There is a strong interest in controlling the speed of light for a variety of optical applications. Slow light has been realized in atomic gas and other systems. Slow light by geometric confinement is more suitable for integrated optical circuits. Various waveguide structures have been proposed and some have been realized. A figure of merit for these pulse delay devices is the delay bandwidth product (DBP). So far the achieved DBP is about 80. Though very large DBP ~10^4 has been achieved by using light-matter interactions in Bose–Einstein condensates, they are not ideal for integrated optics. Recently Tsakmakidis et al. proposed a planar waveguide to stop light by using a negative-index material (DNM) as core layer cladded with a plasmonic metal, with positive index material. A fundamental limitation is that absorptive losses, which are always present when a gain is absent, will destroy the zero group velocity condition and severely limit the achievable performance.

Here we propose a slow light waveguide consisting of an active dielectric core layer cladded by single-negative (Re $\varepsilon < 0$ or Re $\mu < 0$) material. We have discovered a specific window of material parameter conditions in which this waveguide will support guided modes with zero group velocity in case of zero dissipation. The presence of loss will render the group velocity being finite. However, gain can be incorporated into the core layer which can compensate the loss and reduce group velocity. At a critical gain value, $\nu_g = 0$ can be recovered.

We first analyze the basic structure consisting of a symmetric planar waveguide of thickness $d$ made of a dielectric core layer $\varepsilon_D > 0$ cladded with a plasmonic metal (Fig. 1). This waveguide supports both transverse electric (TE) modes and transverse magnetic (TM) modes. An important quantity for the waveguide is the ratio $\sigma = -\varepsilon_M/\varepsilon_D$ which depends on the operating wavelength $\lambda$. Here, for simplicity of analytical discussion, we assume that $\sigma$ is real and also ignore the dispersion of the dielectric. For the TM modes in a symmetric planar waveguide we considered in the paper, the eigenmode equation is $\sqrt{\varepsilon_D k_0 d} f_{TM}(\xi) = f_{TM}(\xi) = (1 + \xi^2 [m \pi - 2 \arctan(\xi/\sigma)]/[1 + \xi^2]$. Here $k_0$ is the wave number in the vacuum and $\xi$ is a free parameter with range $\xi \approx \sqrt{\sigma}$ for supporting guided modes in the waveguide. In terms of $\xi$, one has the transcendental equation $k_x^2 = \sqrt{\varepsilon_D - \varepsilon_M k_0^2}/[1 + \xi^2]$ and $k_z^2 = \sqrt{\varepsilon_D - \varepsilon_M k_0^2}/[1 + \xi^2]$ since $k_x^2 - k_z^2 = (\varepsilon_D - \varepsilon_M)k_0^2$ and $k_x^2 < 0$. As illustrated in Fig. 1(a), the modes propagate along the $z$-direction with longitudinal wavevector component $k_z$. The condition for the waveguide to support degenerate TM modes can be shown to be $\sigma^{1/2} - \nu g^{1/2} = m \pi/2$. One has $\sigma < 1.3510$ for TM modes and $\sigma < 0.2430$ for TM modes [Fig. 1(b)]. The TM modes can exist in ultrathin waveguide and can also be used for trapping light. Using the parameter $\xi$, one has the phase refractive index $n_p = k_z/k_0 = \sqrt{\varepsilon_D - \sigma}/[1 + \xi^2]$. The group refractive index is $n_g = n_p - [1/(2(1 + \xi^2))]\nabla f_{TM}/(d \partial k_0/d \eta)$. The vanishing of $\partial f_{TM}/(d \partial k_0/d \eta)$ will be achieved when $\sigma = 1$.
lead to the divergence of group index $n_g$. If the condition $\sigma < 0.2430$ is fulfilled, there exists a critical thickness $d_c$ for the TM$_2$ modes such that for if $d > d_c$, the waveguide support two modes of the same symmetry, one is a forward-wave and the other one is a backward-wave mode, which carry total energy flow with opposite signs.\textsuperscript{23} For $d < d_c$, the waveguide supports only decay modes. At the critical thickness, these two modes become a single one, the total energy flow is zero, resulting in zero group velocity. This is also due to the negative Goos–Hänchen lateral shifts at the interface between single-negative medium and positive index medium.\textsuperscript{24}

For a fixed thickness $d$, there exists a corresponding critical wavelength $\lambda_c$ at which $v_g = 0$.

As an example, we consider a planar waveguide with $\varepsilon_D = 12.12$ and $\varepsilon_M = -1$ at $\lambda = 1.55$ $\mu$m. Dispersion and loss were incorporated by using a Drude model in all the calculations in this paper.\textsuperscript{25} As shown in Fig. 1(a), for the first branch of the TM modes which have odd-parity with $m = 1$, these modes are all backward waves with negative slope of $n_p$ with respect to waveguide thickness $d$. For $m > 3$, all the TM modes are forward waves. However, for branches with $m = 2$ and 3, the phase index $n_p$ is not a single-valued function of the waveguide thickness, indicating the existence of degeneracy of forward and backward modes. The normalized total energy flow $\langle P_z \rangle$ (Ref. 26) is also plotted in Fig. 1(b). At $\lambda = 1.55$ $\mu$m, there is a critical thickness $d_c = 260$ nm at which the TM$_2$ mode carries zero net energy, $\langle P_z \rangle = 0$ and $v_g = 0$. The sign (+) or (−) of the derivative of $n_p$ with respect to the thickness determines if the wave is a forward- or backward-wave mode, respectively.\textsuperscript{9}

Finite-difference time-domain (FDTD) simulations\textsuperscript{27} show (Fig. 2) that light is stopped in a tapered waveguide. At the width of around 260 nm of the waveguide, we observe a strong yet finite field accumulation. No propagating mode is allowed for $d < d_c$. The phase and group index are calculated and plotted in Fig. 2(b) for the forward-wave modes along the waveguide. We have also simulated the arrival time of a wave packet through a quadratically tapered waveguide.\textsuperscript{25} Inset shows the zoom-in of $n_g$. The cladding metal has $\varepsilon_M = -1 + 0.001i$. The dielectric core layer with $\varepsilon_D = 12.12$ is tapered from 500 nm down to 220 nm over a length of 8 $\mu$m (within the dashed lines). The cladding metal has $\varepsilon_M = -1 + 0.001i$. Inset shows the zoom-in of $n_g$.

![Fig. 2](image-url) (Color online) (a) FDTD simulations of the magnetic field $H_z$ of a continuous wave propagating through a linearly tapered waveguide at $\lambda = 1.55$ $\mu$m. Plotted is the magnetic field intensity. (b) The group index $n_g$. The dielectric core layer with $\varepsilon_D = 12.12$ is tapered from 500 nm down to 220 nm over a length of 8 $\mu$m (within the dashed lines). The cladding metal has $\varepsilon_M = -1 + 0.001i$. Inset shows the zoom-in of $n_g$.

![Fig. 3](image-url) (Color online) The effect of loss and gain on the phase index and group velocity. (a) Without loss, the forward- (TM$_2^f$) and backward-wave (TM$_2^b$) modes merge at the critical thickness with the same phase index, leading to (d) zero group velocity. (b) When the loss is present, the TM$_2^f$ and TM$_2^b$ modes are separated, leading to (e) finite group velocity. (c) When gain $G = -4\pi \text{Im} \varepsilon_D/\lambda$ is introduced and tuned to the critical value, the TM$_2^f$ and TM$_2^b$ modes merge again at the critical thickness, leading to (f) the resorting of zero group velocity. (g) The effect of gain on the group velocity of the TM$_3$ mode in a uniform waveguide of thickness $d = 260$ nm at $\lambda = 1.55$ $\mu$m. The waveguide has $\varepsilon_D = 12.12$ and the cladding Drude metal has $\varepsilon_M = -1 + 0.001i$ at $\lambda = 1.55$ $\mu$m. The critical gain is $G_c = 197.07$ cm$^{-1}$. (h) Critical gain for different absorptive loss $\text{Im} \varepsilon_M$. 

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the Ohmic loss $\text{Im} \ v_M$ in the cladding since the loss in the dielectric core layer can be negligible. The presence of loss splits the mode structure as shown in Fig. 3(a), prevents the group velocity from approaching zero [Fig. 3(d)].

To compensate absorptive loss, we introduce gain in the dielectric core layer by using active medium such as a semiconductor optical amplifier. In the lossless case, the backward- and forward-wave modes join together at the critical point and the group index diverges. However, the gain $G = -4\pi \text{Im}(\varepsilon_r) / \lambda$ in the core layer can compensate for the loss for modes and can furthermore make the phase index $\eta_p$ become real for the propagating modes, thus forcing the forward- and backward-wave modes to recombine. The effect of gain on the phase index and group velocity is shown in Figs. 3(a)–3(g), which shows that the fine-tuning of the core layer gain can recover zero group velocity $v_g = 0$ in the composite waveguide structure. For a fixed amount of loss $\text{Im} \ v_M$ in the cladding layer, there exists a critical gain $G_c$ such that $v_g \propto [G - G_c]^{1/2}$ as shown in Fig. 3(g). For different realistic losses in the cladding ranging from 0.001 to 0.3, different gain values in the core layer are required to compensate the losses and maintain a divergent group index as shown in Fig. 3(h). The introduction of gain can enhance the propagation length of modes whose wavelengths are away from the critical wavelength. The real parts of $\eta_i$ and $\eta_p$ are not sensitive to the amount of gain in the core layer for wavelength away from the critical wavelength. However, the imaginary part of the phase index $\eta_p$ is sensitive to gain, which determines the decay length.

To make use of this waveguide, one must use wavelengths away from the critical wavelength. Very large $\eta_p$ can be obtained for a very wide bandwidth in the present device, which is much larger than that of coupled-resonator optical waveguide optical buffers. The delay time can be very long if these modes can have a very long decay length, which in turn is determined by the imaginary part of the phase index $\eta_p$. For our waveguide the DBP is $\Delta G / \lambda = \Delta n / (4\pi \text{Im} \ v_M)$, where $\Delta n$ is the difference of $\eta_p$ within the bandwidth. The maximum wavelength length $L$ over which the wave will survive is related to the imaginary part of $\eta_p$. For our waveguide, one has $\Delta n \approx 1$, thus one can have DBP $\approx 79 \times 600$ with $L = 12 \text{ cm}$. For $L = 1.2 \text{ cm}$ and DBP $\approx 8000$. Thus very large DBP can be obtained for our waveguide.

Due to the ability to stop light with nonzero phase index, the effective impedance of the waveguide is finite. Thus light can be easily coupled to and from the slow-light waveguide. The present structure as well as that in Ref. 7 has the significant advantage of complete coupling of light with minimal reflections.

Similar results are obtained for the stopping of TE modes if we consider negative permeability materials (Re $\mu_M < 0$ and Re $\varepsilon_M > 0$) for cladding of a dielectric core layer. Since only single-negative materials are needed, our proposed scheme to slow and stop light can be realized for a broad range of frequencies. For claddings with metallic materials, the key is to have Re($-\varepsilon_M / \varepsilon_p$) $< 0.243$ to stop TM modes. In the infrared, one can use silicon as the core layer and various polar materials with negative permittivity for cladding. In the visible, we can use noble metals for the cladding layer. At microwave and terahertz frequencies, many metamaterials can be engineered to exhibit negative permittivity, such as a metallic wire mesh structure. Negative permeability materials such as ferrites or split-ring resonator arrays can be used to stop TE waves.

In this paper, we have provided a complete framework for the analysis of the modes in a planar active dielectric waveguide cladded by single-negative metamaterials. Our results can be readily tested in experiments. The proposed ideas here have important potential for applications in future generation optoelectronics.

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