Comment on "Wave Refraction in Negative-Index Materials: Always Positive and Very Inhomogeneous"

Valanju, Walser and Valanju (VWV) [1] have shown that for a group consisting of two plane waves incident on the interface between a material of positive refractive index (PIM) and material of negative refractive index (NIM), the group velocity refracts positively. Here we show that this is true only for the special two plane wave case constructed by VWV, but for generic localized wave packets, the group refraction is generically negative.

The sum of two plane waves of wavevector and frequency $(\mathbf{k}_1, \omega_1)$ and $(\mathbf{k}_2, \omega_2)$, considered by VWV, can be written as $2e^{i(\mathbf{k}_0\cdot\mathbf{r}-\omega_0 t)}\cos[(1/2)(\mathbf{\Delta k}\cdot\mathbf{r}-\Delta\omega t)]$, where $(\mathbf{k}_0, \omega_0)$ the average wavevector and frequency and $(\mathbf{\Delta k}, \Delta\omega)$ denote their differences. Clearly, the argument of the cosine is constant along planes, which propagate in time along the direction of their normal, $\mathbf{\Delta k}$. We have carried out numerical simulations of wave packets incident on the PIM-NIM interface and for the 2-wave case arrive at conclusions similar to VWV. For arbitrary number of incident plane waves whose $\mathbf{k}$ vectors are all parallel, the group refraction is again positive. Note that in all these special cases the packet remains nonzero on infinite planes.

Here we show that for any wave packet that is spatially localized, the group refraction is *generically negative*.

For 3 (or more) waves whose wave vectors not aligned, the group refraction will be negative. For example, consider three wave vectors in PIM in the x-z-plane, whose magnitudes are, $k-\delta k$, k, $k-\delta k$ and whose angles with the z-axis are, $\theta-\delta\theta$, θ, $\theta+\delta\theta$, respectively. The dispersion $k^2=(\omega^2-\omega_p^2)(\omega^2-\omega_b^2)/c^2(\omega^2-\omega_0^2)$ were used for NIM. The results are shown in Fig. 1. The wave packet refracts negatively, in obvious contrast to VWV. As we have seen, two plane waves result in a wave packet-like structure which is constant along planes; the addition of a third wave breaks the planes into localized wave packets which refract negatively.

A packet constructed from a finite number of plane waves will always give a collection of propagating wave pulses, as seen in Fig. 1. A wave packet localized in one region of space, as occurs in all experimental situations, can only be constructed from a continuous distribution of wavevectors. Consider such a wave packet incident from outside the NIM, $E=E_0\int d^2k f(\mathbf{k}-\mathbf{k}_0)e^{i(\mathbf{k}\cdot\mathbf{r}-\omega(\mathbf{k})t)}$, where $\omega(\mathbf{k})=ck$. If $f(\mathbf{k}-\mathbf{k}_0)$ drops off rapidly as $\mathbf{k}$ moves away from $\mathbf{k}_0$, $\omega(\mathbf{k})$ can be expanded in a Taylor series to first order in $\mathbf{k}-\mathbf{k}_0$ to a good approximation. This gives, $E=E_0 e^{i(\mathbf{k}_0\cdot\mathbf{r}-\omega(\mathbf{k}_0)t}g(\mathbf{r}-ct\mathbf{k}_0/k_0)$, where $g(\mathbf{R})=\int d^2k f(\mathbf{k}-\mathbf{k}_0)e^{i(\mathbf{k}-\mathbf{k}_0)\cdot\mathbf{R}}$. Inside the NIM, $\mathbf{k}$ and $\mathbf{k}_0$ in the argument of the exponent get replaced by $\mathbf{k}_r$ and $\mathbf{k}_{r0}$ which are related to $\mathbf{k}$ and $\mathbf{k}_0$ by Snell's law. Then the wave packet once it enters the NIM is given by

$$E_r = E_0' e^{i(\mathbf{k_{r0}}\cdot\mathbf{r}-\omega(\mathbf{k_{r0}})t}g_r(\mathbf{r}-\mathbf{v}_{gr}t), \quad (1)$$

where $g_r(\mathbf{R})=\int d^2k f(\mathbf{k}-\mathbf{k_0})e^{i\mathbf{R}\cdot[(\mathbf{k}-\mathbf{k_0})\cdot\nabla_{\mathbf{k}}\mathbf{k_r}]}$. Here $\mathbf{k}_{r0}$ denotes $\mathbf{k}_r$ evaluated at $\mathbf{k}=\mathbf{k}_0$ and $\mathbf{v}_{gr}=\nabla_{\mathbf{k}_r}\omega(\mathbf{k}_r)$ evaluated at $\mathbf{k}_r=\mathbf{k}_{r0}$. Thus, the refracted wave moves with the group velocity $\mathbf{v}_{gr}$. Evaluation of Eq. (1) for a Gaussian wave packet shows that the incident packet gets distorted but the maximum of the packet moves at $\mathbf{v}_{gr}$. For the case of an isotropic medium, considered by VWV [1], the group velocity is anti-parallel to the wave vector in the medium. Hence, the group velocity will be refracted the same way as the wavevector is, contrary to the claim of VWV [1].

Thus VWV's statement that the "Group Refraction is always positive" is true only for the very special (and rare) wave packets constructed by them and is incorrect for more general wave packets that are spatially localized.

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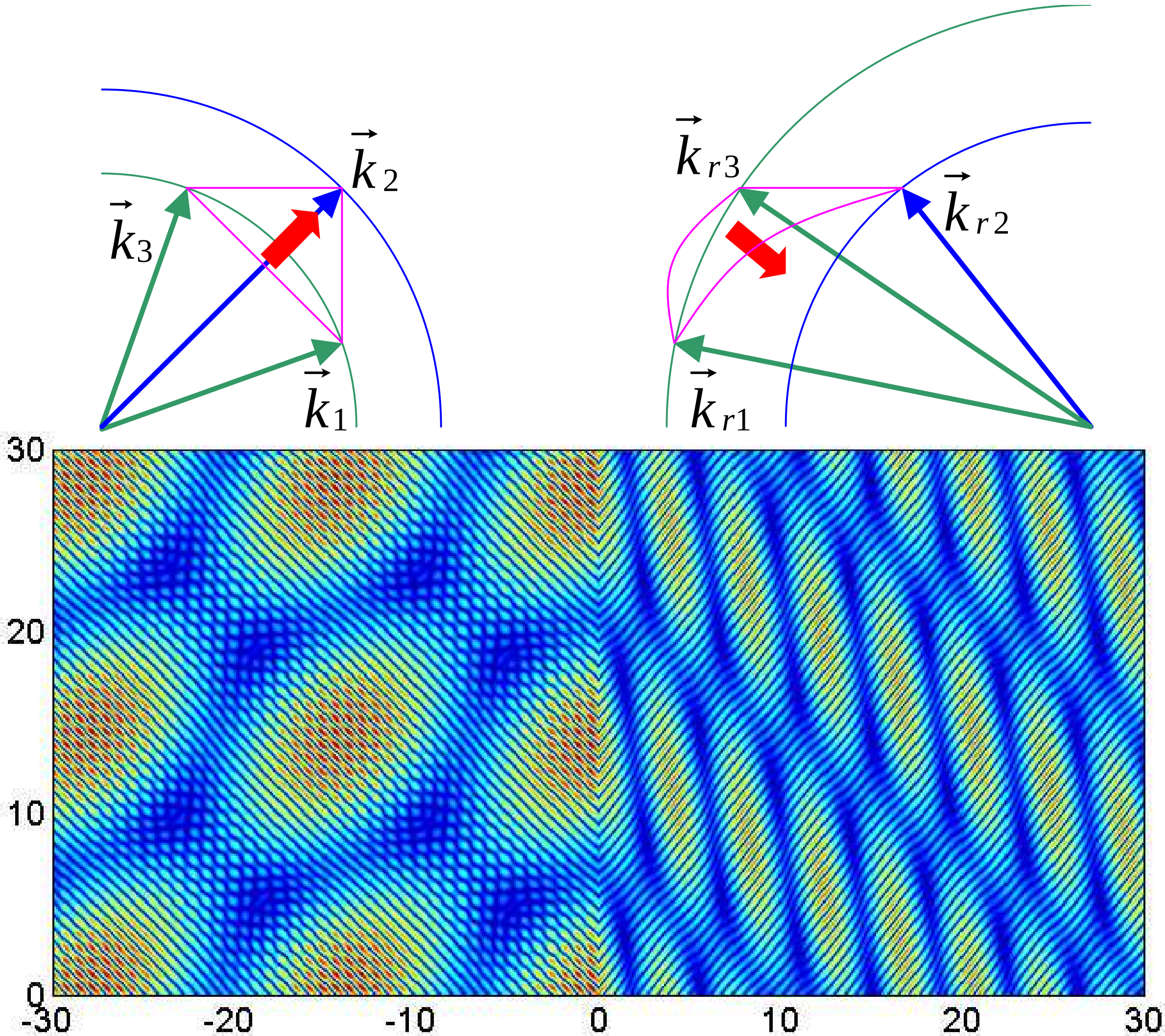
W. T. Lu, J. B. Sokoloff and S. Sridhar

Department of Physics, Northeastern University, 360 Huntington Avenue, Boston, MA 02115.

[1] P. M. Valanju, R. M. Walser and A. P. Valanju, Phys. Rev. Lett. 88, 187401 (2002).

FIG. 1: Negative refraction of 3 plane waves with $k=6.32$, $\delta k=0.32$, $\theta=\pi/4$, $\delta\theta=\pi/60$, $\omega_0=2$, $\omega_b=8$, $\omega_p=10$, and $c=1$. (Inset above) Wavevectors for the 3 plane waves in the PIM (left) and NIM (right). The thick arrows indicate the wave packet propagation direction.



$\vec{k}_2$
$\vec{k}_3$
$\vec{k}_1$
$\vec{k}_{r3}$
$\vec{k}_{r2}$
$\vec{k}_{r1}$
30
20
10
0
-30
-20
-10
0
10
20
30