Computer-aided modeling of superconducting striplines with ground planes using critical state models

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(Received 24 April 2001; accepted for publication 29 May 2001)

The effect of ground planes on the current-induced critical states and flux penetration for a thin superconducting strip carrying a microwave current has been calculated numerically. The nonlinear response represented by the dependence of the surface resistance and reactance were also calculated. We find that when the distance d between the ground planes is smaller than the width a of the strip, i.e., $d \leqslant a$, the ground planes affect the nonlinear response significantly. In particular we study the effects of ground planes placed at a distance $d=a/5$. We compare and contrast these effects with the results of similar models without the ground planes. The ground planes reduce the current crowding at the film edges, thus reducing the nonlinearities arising from critical state flux motion. The numerical procedure can be incorporated into computer-aided design of superconducting microwave circuits. [DOI: 10.1063/1.1389339]

I. INTRODUCTION

High temperature superconductors have the potential to replace normal metals for passive microwave structures due to their extremely low surface resistance.[1–4] A key feature of the high temperature superconductors is the inherent nonlinear response at microwave frequencies, which is present over a wide range of input powers.

The current-driven critical state model[4] has proven to be particularly useful not only because it fits experimental measurement of the surface resistance R_S over a wide range of temperature and power levels,[5] but also because its usefulness extends to modeling hysteretic losses in superconducting transmission lines.[6]

Unfortunately these models can be worked out analytically only for very idealized geometries,[4] and most existing computer-aided design (CAD) software does not take into account the inherent nonlinearities of superconductors. Thus device design using hard superconductors demands numerical handling of these nonlinearities. Numerical solutions for nonlinearities based on Ginzburg–Landau theory have already been formulated and analyzed.[7,8]

In hard superconductors, the presence of lattice defects prevents free entry or exit of vortex lines, and the observed magnetization is highly irreversible and dependent on the specimen size. For exactly the same reason, the hard superconductors can sustain large transport currents in contrast to the ideal type II superconductors. To relate these two aspects quantitatively, Bean[9] proposed the well known "critical-state" model. In this model, the hard superconductors are treated as a perfect conductor until the current density J induced in the specimen reaches a certain critical current density J_c. The critical state is specified by the condition $J < J_c$ everywhere.

The critical state model was first solved by Bean for the geometry of a semi-infinite slab where the field was applied parallel to its face. The geometry where the field is applied perpendicular to a thin strip was solved relatively recently by several groups and the field and current distribution[10,11] and the surface impedance from hysteretic loss[4] was obtained analytically.

Our aim is to develop a numerical paradigm that can be used for designing devices using hard superconductors. In this article we demonstrate that such solutions are possible[6] and are conveniently extendable by adding additional circuit elements into the system. We solve a system consisting of an infinite superconducting strip surrounded by ground planes made of perfect conductors on both sides of the superconducting strip, including flux penetration due to a current-induced critical state in the superconducting strip. We obtain the flux penetration depth, current and field profiles, and surface impedance of this system and compare them with the scenario with no ground planes.[6] We observe that in general, the presence of ground planes reduces the flux penetration depth and hence the surface impedance of the striplines. Thus we are able to successfully treat a nearly realistic geometry within the critical state model for which analytic solutions are not available and are expected to be impossible.

II. SUPERCONDUCTING STRIPLINES

Consider a stripline with two ground planes of infinite width at a distance d from the central conductor. These ground planes together will then carry a current in the opposite direction and hence complete the circuit. The geometry of the center superconductor, the ground planes, and our choice of axes are shown in Fig. 1.

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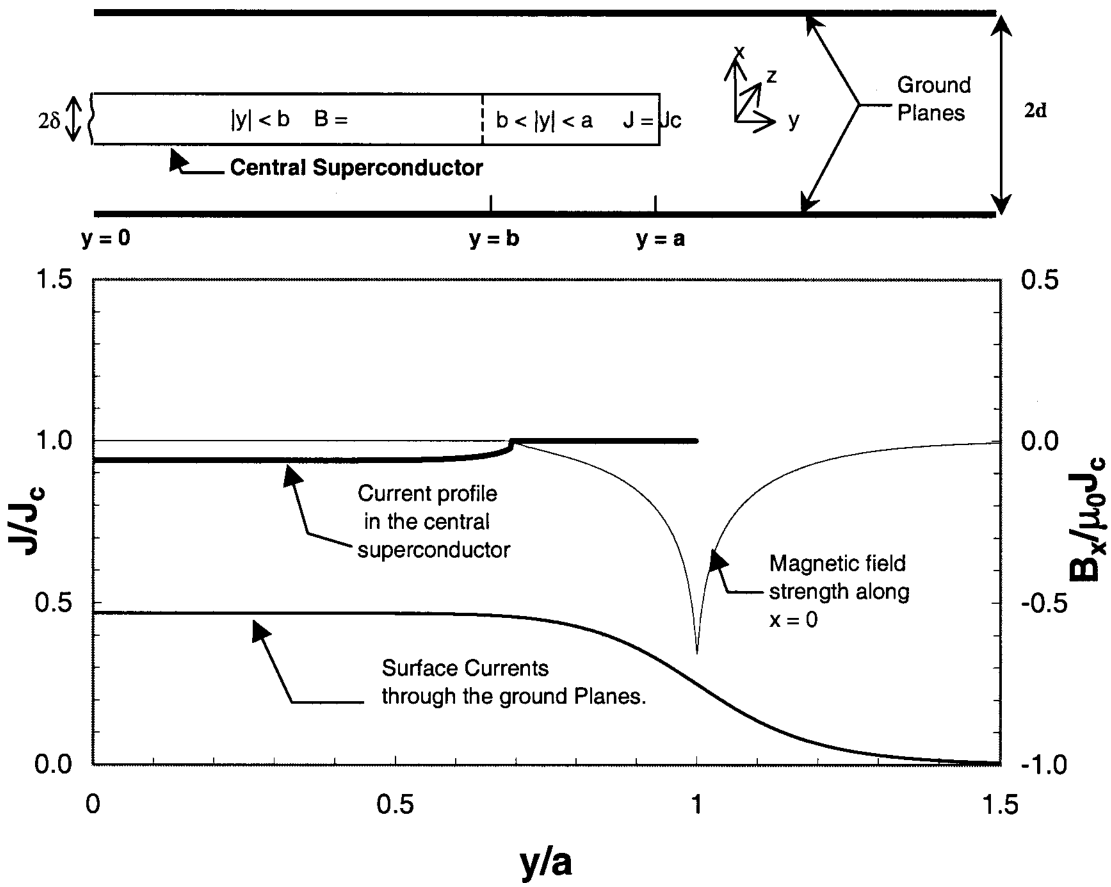

FIG. 1. (Top) Cross section of the circuit system consisting of the central superconducting strip and the two ground planes surrounding it. (Bottom) Field B and current J profiles in the central conductor and in the ground planes for $b/a=0.8$.

The generic critical model we intend to solve assumes:

(1) $|J(x,y)|\leqslant J_c$ where $J(|x|>b)=J_c$ with J_c being the critical current density.

(2) The magnetic induction $\mathbf{B}(x=0,y)=0$ for $|y|\leqslant b\leqslant a$.

Further, the current density averaged over the thickness $\mathbf{J}=J(y)\hat{k}$ where $\hat{k}$ is a unit vector along the z axis.

We numerically investigate this model along with the ground planes. The effects of ground planes depend on their distance from the central conductor. We find that the effects of ground planes on current patterns are small (<0.1%) for $d\geqslant 1.5a$ but are significant for $d<a$. For this reason we chose $d=0.2a$, so we can study the influence of ground planes. Interestingly, we find that ground planes amplify the nonlinearity in these kind of models.

We emphasize that conditions (1) and (2) lead to a hysteresis law that is unchanged under the addition of the ground planes. As expected, the current profiles in the ground planes also display hysteretic patterns due to the presence of the superconducting central conductor.

The problem with the striplines that we consider here can be specified by the following conditions.

(1) Inside the superconducting strip of width $2a$ and thickness 2δ ($|y|\leqslant a, |x|\leqslant\delta$), we have

$$J=J_c \quad \text{for } b\leqslant|y|\leqslant a, \qquad B_x=0 \quad \text{for } x=0 \quad \text{and } |y|<b. \tag{1}$$

(2) In the region between the ground planes and outside the central superconductor, there is no current:

$$J=0 \quad \text{for } \delta<|x|<d \quad \text{and } |y|>a. \tag{2}$$

(3) At the surface of the ground planes at $|x|=d$ we have the standard boundary conditions for a perfect superconductor:

$$B_x(|x|=d)=0 \quad \text{and } B_y(|x|=d)=\mu_0 J_s, \tag{3}$$

where J_s is the surface current in the ground planes.

Due to the rectangular symmetry of the problem one needs to solve this problem only in the first quadrant. We use the standard simultaneous over reduction routine to find the solution of the equation $\nabla^2\mathbf{A}=-\mu_0\mathbf{J}$ over the region $0<x<d$ and $0<y<a$ with the boundary conditions on $\mathbf{A}=A(x,y)\hat{k}$ given by

$$A(d,y)=A(x,a)=0, \qquad A(-x,y)=A(x,y)=A(x,-y). \tag{4}$$

One can then use an iterative algorithm similar to the one described in Ref. 6 with the additional boundary condition Eq. (4) to solve for current profiles and magnetization inside the superconductor. The surface current through the ground planes is then given by

$$J_s(y)=-\mu_0^{-1}\partial_x A(x,y)|_{x=d}. \tag{5}$$

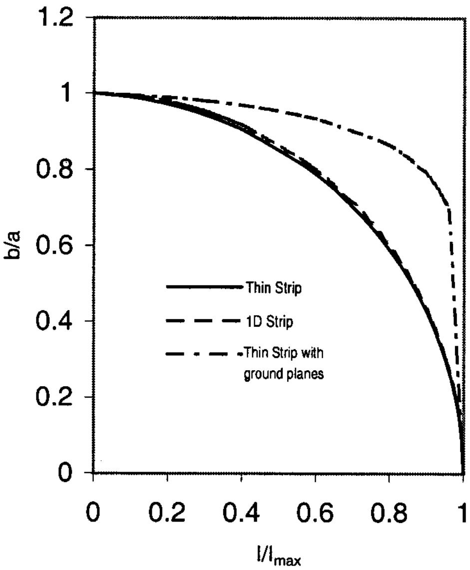

FIG. 2. Length of flux-free region (b/a) in three different scenarios. Superconducting strip with ground planes for $d/a=0.2$ (dot-dash line), analytical solution (Ref. 11) for a strip of zero thickness without ground planes (dashed line), and numerical solution of a thin stripline without ground planes (solid line).

We calculate the flux penetration depth $\Lambda/a \equiv 1-b/a$ for the current-carrying strip. In Fig. 2 we display the flux free region $(1-\Lambda/a)=b/a$ as a function of the applied current $I/I_{\max}$ for $\delta/a=1\%$ and compare that with the results for the no-ground plane scenarios.

Figure 3 shows the corresponding current and field distributions. One can see from Figs. 2 and 3 that the penetration law becomes more nonlinear and the current density in the field free region increases as the spacing between the stripline and the ground planes decreases. These results are expected, since the ground planes force the normal component of **B** to be very small (near zero) on the surface of the strip. These results can then determine the current critical states and the corresponding magnetic field distribution for ac currents with a peak current of I_0.

Any Bean-like model, with a critical current J_c such that $J \leqslant J_c$ everywhere, will follow the specific Hysteresis law[6]

$$J_{\downarrow}(y,I,J_c)=J_{\max}(y,I_0,J_c)-J_{\uparrow}(y,I_0-I,2J_c). \tag{6}$$

This is because, when the current is reversed, after reaching the peak I_0, the current density starts to decrease from the edges with an effective critical current density in the opposite direction of $2J_c$. In Fig. 4 we display the field and current profiles in the central conductor and the ground plane for a complete cycle with a peak transport current $I_0/I_{\max}=0.79$: The transport current I_T is given by $I_T=I_0\cos(\omega t)$.

We further investigate the effects of ground plane on the resistance R' and the reactance X' per unit length. This can be calculated numerically using[11,6]

$$R'=\frac{R}{l}=\frac{2\nu}{J_c\delta}\left(\frac{I_{\max}}{I_0}\right)^2\int_1^{-1}d\left(\frac{x}{\delta}\right)\int_{b(I_0)/a}^{1}d\left(\frac{y}{a}\right)\times\int_{b(I_0)/a}^{y/a}d\left(\frac{y'}{a}\right)B_x\left(\frac{y'}{a},\frac{x}{\delta},t=0\right), \tag{7}$$

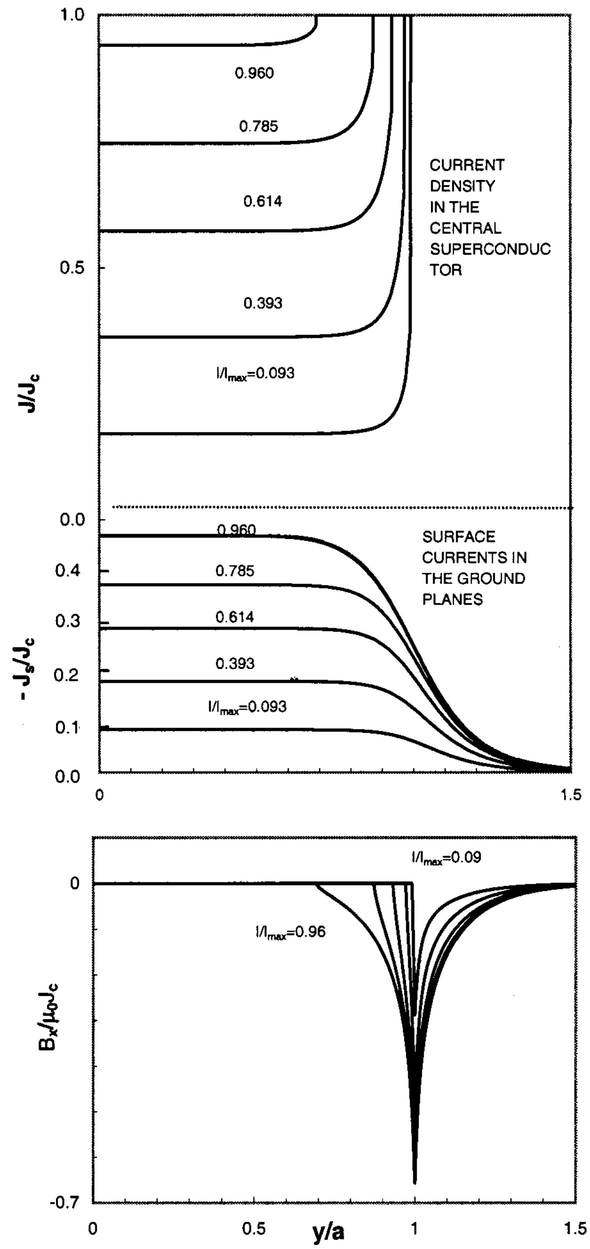

FIG. 3. The current profile (top) in the central superconducting strip, (middle) in the ground planes, and (bottom) the field profiles at the center conductor location, for $I/I_{\max}$=0.093, 0.393, 0.614, 0.785, and 0.960.

$$X'=\frac{X}{l}=\frac{\pi\nu}{J_c^2 a\,\delta\mu_0}\left(\frac{I_{\max}}{I_0}\right)^2\int_{-\infty}^{\infty}d\left(\frac{y}{a}\right)\times\int_{-\infty}^{\infty}d\left(\frac{x}{\delta}\right)B^2\left(\frac{x}{\delta},\frac{y}{a},I_0\right). \tag{8}$$

Figure 5 displays the surface reactance R' and X' as a function of the peak current, $I_0/I_{\max}$. Interestingly these graphs show that the presence of the ground planes scales down the value of both R' and X' by a constant factor for all values of $I_0/I_{\max}$.

III. CONCLUSION

In this article we present the exact effects of ground planes on the microwave response of a superconducting

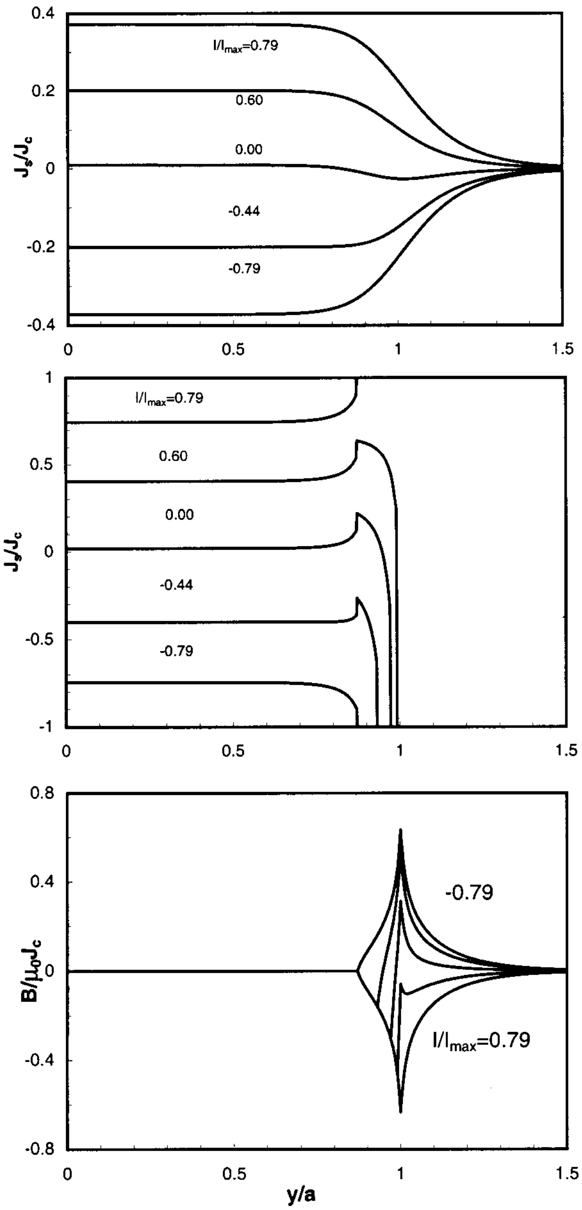

FIG. 4. The field and current profiles in the central conductor and the ground plane for a half cycle from $I_0/I_{max}=0.79$ to $I_0/I_{max}=-0.79$. (Top) Surface currents in the ground planes. (Middle) Current profile in the center of the central conductor. (Bottom) Field profiles in the central conductor.

stripline using numerical calculations. The results, which are valid in the limit of strip thickness $\delta \ll$ strip width a, approach the numerical results of Ref. 6 and the analytical results[4] as we move the ground planes to infinity. We find that for a separation $d/a=0.2$, the presence of the ground planes leads to strong effects on the penetration law of the critical state model. The ground plane reduces the current crowding at the film edges, because the ground planes tend to spread out the currents in the film. This minimizes the losses due to the critical states for small values of current (I/I_{max}), and correspondingly reduces the surface resistance by an order of magnitude. Interestingly though, the shape of

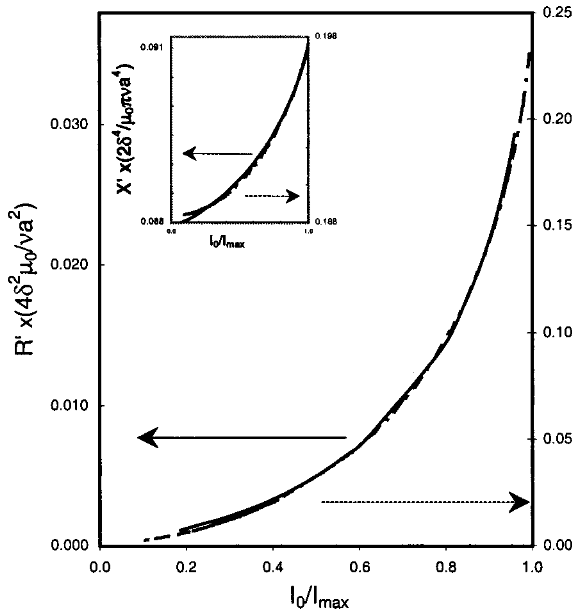

FIG. 5. Dependence of resistance R' and reactance (X') on total current for a system of stripline with ground planes (solid lines, left axis), and without the ground planes (dashed lines, right axis).

the surface resistance (reactance), $R_s(X_s)$ vs I/I_{max} plot does not change significantly, and is effectively scaled down by a nearly constant factor, confirming that the results of Ref. 4 hold true even in the presence of ground planes.

The calculations and results presented here demonstrate that numerical calculations are feasible for realistic device structures using the hysteretic critical state model. This approach can be implemented into a computer-aided design framework for the design of passive superconducting microwave components.

ACKNOWLEDGMENTS

This work was supported by NSF-9711910 and AFOSR-F49620-98-1-0021. This work benefited from the allocation of time at the Northeastern University Advanced Scientific Computing Center (NU-ASCC).

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