Microwave 2-Disk Scattering

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Abstract

Microwave scattering experiments on N-disks systems were done in the regime where the resonances were dominated by the time scale a classical particle can stay in the repeller region. In this paper we present data for a 2 disk system. Data as a function of disk separation was obtained and direct evidence of the single periodic orbit in the raw signal was found. Good agreement is found with semi-classical theory.

It has been known for some time, that the stationary state properties of a time independent wave mechanical system reflect the corresponding classical mechanics [1]. In closed systems, such as billiards with convex boundaries, the eigenvalues and eigenfunctions show universal signatures of chaos which depend only on the symmetry of the Hamiltonian. For example, the spacing of eigenvalues show level repulsion which are in agreement with Random Matrix Theory (RMT). A similar universality should exist for open systems. Much progress has been made using an S-matrix formalism connecting chaotic scattering with RMT [2, 3, 4]. Using periodic orbit theory and a reordering of the periodic orbits (POs), Cvitanović and Eckhardt [5] were able to compute the semi-classical resonances of the three disk system which compared very well with the calculations of Gaspard and Rice [6].

Microwave cavity systems are a versatile experimental tool to further our understanding of Quantum Chaos. In closed systems, such as the Sinai billiard cavities, these experiments have lead to the experimental observation of scars [7] and precision tests of eigenvalue statistics [8]. In any real experimental situation one always has leads coupling into the system. So in fact a cavity experiment can also be thought of as a scattering system. This was exploited by Doron et al [9] in their experiment on an elbow shaped cavity, in which they looked at the reflection of microwaves, and analyzed it using a scattering theory approach. It was found that the width of the resonances could be explained by absorption in the cavity walls.

In this paper we study experimentally, microwave scattering from 2 disks in the regime where the transmission resonance widths are dominated by the *openness* of the system. It is well known that the classical motion in the 2 disk system is integrable and has been studied in detail by José et al [10]. For $N > 2$, the classical motion is chaotic and the scattering function is a non-differentiable function, which has a sensitive dependence on initial conditions.

The experiment consists of two parallel planes of copper between which, electromagnetic fields are set up. Below a certain frequency, which is inversely

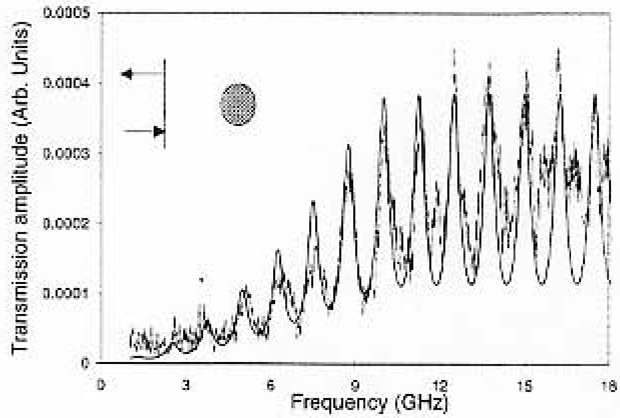

Figure 1: Transmission amplitudes as a function of frequency for the geometry shown in the inset. The arrows correspond to the input and output coupling positions. The thick line is a comparison to semi-classical theory. Minor discrepancies are observed, which may correspond to modes in the leads.

proportional to the distance of separation of the copper planes, the electric field perpendicular to the planes (z direction) can be written as:

$$(\nabla^2_{x,y} + k^2)E_z = 0 \tag{1}$$

Further boundary conditions, $E_z = 0$, can be imposed by placing conductors at the appropriate places. Therefore by imposing Drichlet boundary conditions on a disk and along the symmetry axis of the N-disk system, one can construct a geometry which looks at one of the symmetries of the N-disk system. In the experiment, copper is used to define the reflecting boundaries, and microwave absorbers, to define the directions in which the waves are scattered to infinity. Measurements are made in the transmission mode by inductive coupling in two coaxial cables. Experiments which mapped to the two disk geometry were done utilizing only one disk and the symmetry properties of the scattering geometry. Figure 1 (inset) shows the geometry. The transmission data was taken, recording both, the amplitude and the phase. The obtained resonances correspond to the A_2, antisymmetric poles of the scattering process [11]. The magnitude of the transmission as a function of frequency is plotted in figure 1. One observes broad resonances, which have widths much larger than that observed for closed

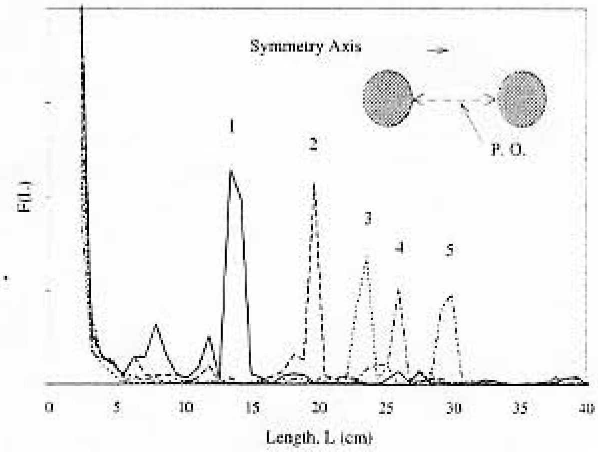

Figure 2: Fourier transform of the transmission data for different disk-wall separations. A single peak due the only periodic orbit is seen. Inset: Corresponding PO of the 2 disk system.

systems.

A semi-classical analysis for the 2 disk geometry can be easily done as there is only one periodic orbit. The scattering resonances are obtained by using the formula

$$k_n = \frac{2\pi n + i\frac{1}{2}\ln(\Lambda)}{(R-2a)} \qquad n = 1, 2, 3... \tag{2}$$

where R is the distance of separation of the disk centers, a is radius of the disks [11]. Λ corresponds to the instability of the periodic orbit bouncing between the two disks and is given by

$$\Lambda = \frac{R - a + \sqrt{R^2 - 2Ra}}{a}$$

The expression for k_n is complex, with the imaginary part corresponding to the life time a particle stays in the geometry. In figure 1 the scattering function for a geometry corresponding to $R = 34$ cm and $a = 5$ cm is plotted as a function of frequency, f, along with the semi-calculation using a Lorenzian formula

$$T(f) = \sum_n \frac{A_i}{(f - f_{nR})^2 + f_{nI}^2} \tag{3}$$

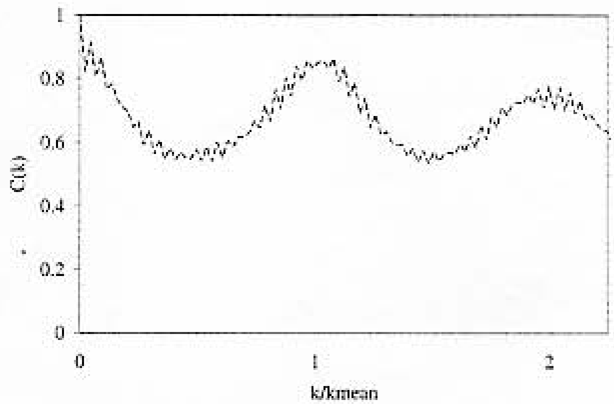

Figure 3: Correlation in the frequency spectrum show non-universal behavior.

where A_i the amplitudes, were adjusted to correspond to the experimental values, and $f_{nR}+if_{nI} = \frac{c}{2\pi}k_n$, where c is the speed of the electromagnetic wave. One sees very good agreement not only with the locations of the resonances, but also the widths. This shows that the resonances widths are due to the finite escape rate of a classical particle from the periodic trajectory. Some deviations are seen from this form and may be attributed to modes in the leads which appear as weak peaks.

Analysis using Fourier transforms (FT) and correlations of the trace was done. The FT of the data is shown in figure 2. A single peak is seen which corresponds to the only primary PO in the system which is along the line connecting the disk center with the symmetry axis. To further verify the connection to the PO, experiments were done with different disk separations and the peak was found to scale accordingly. In figure 2 the peaks marked 1 through 5 correspond to a PO of length 14, 20, 24, 26 and 30 cm respectively.

The system can also be modeled by a many channel scattering approach, where the input and the output can be modeled as two of these channels. The other channels are the ones which couple to waves scattering to infinity. The correlations in the frequency spectrum can be also analyzed using a S-matrix formalism [2, 12]. Of interest is the quantity $C(k) =< S^*(K)S(K+k) >_K$, called the average correlation function which may show a universal signature of chaos in a open system. The quantity $C(k)$ for the 2-disk data has a form which is non-Lorenzian, and shows correlations which can be seen in figure 3. Here k has been rescaled to the average resonance spacing and the data corresponds to $R = 40$ cm.

In conclusion we have demonstrated an experiment in which ideas from semi-classical theory are directly applicable to an open scattering system. Several more interesting issues need to be addressed in further experiments. Improvements in the calibration technique needs to be done to eliminate the resonances due to the leads. Diffraction is an important effect in scattering of waves from hard boundaries. Indeed this issue has already been studied theoretically and seen to give rise to additional weak resonances [13]. As discussed earlier, the experiments are easily extended to N-disk systems which display chaos. Also the electric field distribution inside the region can be measured by directly probing their amplitudes. Future work will concentrate on these aspects of microwave scattering.

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