APPLIED PHYSICS LETTERS **89**, 084104 (2006)

Negative refraction and plano-concave lens focusing in one-dimensional photonic crystals

P. Vodo,[a] W. T. Lu,[b] Y. Huang, and S. Sridhar[c]
Department of Physics, Northeastern University, Boston, Massachusetts 02115
and Electronic Materials Research Institute, Northeastern University, Boston, Massachusetts 02115

(Received 20 April 2006; accepted 3 July 2006; published online 25 August 2006)

Negative refraction is demonstrated in one-dimensional (1D) dielectric photonic crystals (PCs) at microwave frequencies. Focusing by plano-concave lens made of 1D PCs due to negative refraction is also demonstrated. The frequency-dependent negative refractive indices, calculated from the experimental data, match very well with those determined from band structure calculations. The easy fabrication of one-dimensional photonic crystals may open the door for future applications. © *2006 American Institute of Physics*. [DOI: 10.1063/1.2338644]

Negative refraction[1] and related phenomena such as flat lens imaging[2–4] and plano-concave lens focusing[5] have attracted a lot of attention in physics and engineering. Negative refraction allows subwavelength imaging[6] and focusing of far field radiation by concave rather than convex surfaces with the advantage of reduced aberration[7,8] for the same radius of curvature. Negative refraction has been realized in two- and three-dimensional structures in metamaterials[9,10] and photonic crystals.[11,12] But negative refraction has not yet been demonstrated in one-dimensional (1D) photonic crystals (PCs).

In this letter we present a study of left-handed electromagnetism in 1D PCs at microwave frequencies. Negative refraction is achieved in the second band of the 1D PCs. Focusing of plane wave radiation by plano-concave lenses made of 1D PCs is also demonstrated. The inverse experiment, in which the lens produces plane waves from a point source placed at the focal length, at the same frequency of operation, is confirmed as well. The frequency-dependent negative refractive indices, calculated from the experimental data, match very well with those determined from band structure calculations. The easy fabrication of 1D PCs may open the door for microwave and optical applications.

The experiments were carried out in a parallel-plate waveguide of height $h=1.25$ cm and size of 3×6 ft^2. For frequency below 12 GHz, the excitation in these quasi-two-dimensional system is the transverse magnetic (TM) modes with the electric field in the vertical direction. The electric field of the microwaves is scanned using a monopole antenna attached to an X-Y robot in the frequency window of $3-11.5$ GHz. An HP-8510C network analyzer is used for measuring the transmission characteristics. A schematic diagram of the experimental setup is shown in Fig. 1.

Alumina bars with permittivity $\epsilon=8.9$ were laid out to form prisms of right angle triangles. The bars have a height $h=1.25$ cm and width $d=0.5$ cm. The refraction experiments were performed on two PC prisms. The first prism (PC1) has a lattice constant $a=1$ cm and incident angle of 45° while the second one (PC2) has $a=0.8$ cm and angle of 51°. All bars have a perpendicular cut instead of a slanted one, as numerical simulations show that a perpendicular cut reduces the modulations of the outgoing waves, partly due to the absence of sharp corners. A plane wave incident normally to one surface is refracted by the hypotenuse of the 1D PC prism as shown in Fig. 1, defined as the surface of refraction.

The 1D PC is a model that is exactly soluble.[13] For the TM modes, it is just the Kronig-Penney model[14] with an energy dependent potential. The equifrequency and band structure for the TM modes for the filling factor $d/a=0.5$ are shown in Figs. 2(a)–2(c). A band gap for normal incidence is located between 5.55 and 8.9 GHz. The second passband is between 89 and 12.7 GHz and has negative refractive index. The refraction of a microwave beam by the PC1 prism at 10.55 GHz is shown in Fig. 1. By fitting the outgoing beam with a plane wave, the wave front is determined and an effective index $n_p=-0.85$ is obtained using Snell's law.

Although positive refraction is predicted for both PC prisms in the first band, the incident angle for each PC prism exceeds the corresponding critical angle, resulting in total internal refraction. The average power of the scanned points is plotted as a function of frequency in Fig. 2(d). While the outgoing signal is very weak for the total internal reflection and band gap frequency regions, an interesting observation is that there is a strong leaking from the prism near the edges of the band gap.

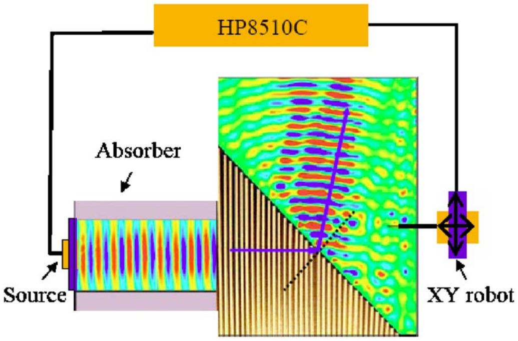

FIG. 1. (Color online) Schematic diagram of the microwave experimental setup and negative refraction of plane waves by the 1D PC1 prism. Angle of incidence is 45° and angle of refraction is −36.9° resulting in negative refraction with refractive index $n_p=-0.85$ at 10.55 GHz. The real part of S_{21} scale: on the left side varies from −0.015 to 0.015, on the right side from −0.006 to 0.006.

---

[a] Electronic mail: vodo.p@neu.edu
[b] Electronic mail: w.lu@neu.edu
[c] Electronic mail: s.sridhar@neu.edu

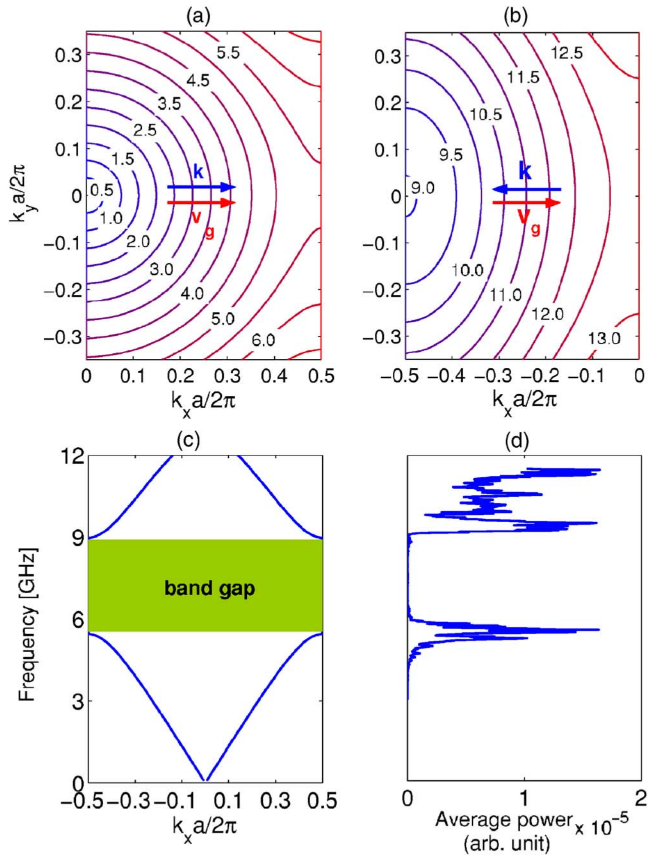

FIG. 2. (Color online) (a) Equifrequency (in GHz) surface (EFS) of the first band and (b) EFS of the second band for the TM modes of the 1D PC with $a=1$ cm, $d=0.5$ cm, and $\epsilon=8.9$. (c) Band structure for normal incidence ($k_y=0$). (d) Frequency vs average outgoing power, calculated as the average of $|S_{21}|^2$ in the scanned area. From 3 to 5 GHz all energy is internally reflected. Note the leaking of energy from around the edges of the band gap. Measurement was performed between 3 and 11.5 GHz.

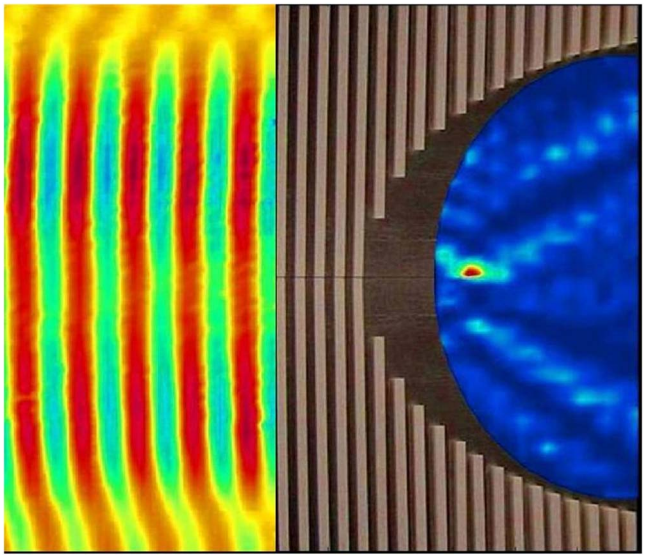

FIG. 3. (Color online) Focusing by a 1D plano-concave PC lens having $d=0.5$ cm, $a=1$ cm, and radius of curvature $R=18$ cm. The focus point observed at 9.5 GHz is 6.15 cm from the concave lens surface. A photograph of the PC is superimposed to obtain the final figure. On the left side, field map of the incoming plane wave is shown (real part of transmission coefficient) and on the right side, intensity of the focus point. Scale: on the left, from −0.03 to 0.03, on the right side from 0 to 4×10^{-3}. Dimensions of the lens are 19×38 cm^2.

The same alumina bars were used to form a plano-concave lens of 1D PCs, with $a=1$ cm. The concave radius of the lens is $R=18$ cm. A sharp focal point is located at 6.15 cm away from the curved surface when a microwave beam incidents at frequency of 9.5 GHz. From left to right in Fig. 3, the incoming plane wave, a real picture of the PC lens and the emerging mapped field are shown. Clear focusing is observed in the frequency range of 9.2–11.5 GHz.

An inverse experiment in which a point source is placed at the observed focal point of the lens at a single frequency is also carried out. As shown in Fig. 4, a circular wave front from the point source after passing through the lens emerges as a plane wave. These two remarkable results validate the behavior of a left-handed plano-concave lens.

The refractive index of the lens can be estimated using the lens equation $n=1-R/f$ which is valid for thin plano-concave and plano-convex lenses in the geometric optics limit. Here R is the radius of curvature and f is the focal length. Using this equation we get $n=-0.85$ at 9.5 GHz for the lens shown in Fig. 3. A real focus by a plano-convex lens is achieved with $n>1$ and $R<0$ while for the plano-concave lens with $n<1$ and $R>0$.

The refractive indices n_p determined from the prism refraction experiments (PC1 and PC2) and the plano-concave lens one are shown in Fig. 5. Very good agreement with those calculated from the band structure is observed. The index n_p determined from the focusing experiment fits better with theoretical results as the frequency is increased. This may be due to the reduced finite-size effect and aberration at higher frequency.

The nature of the left-handed electromagnetism and focusing can be understood from the dispersion characteristics of the 1D PCs. From the equifrequency surfaces shown in Fig. 2(b), it can be deduced that in the second band the wave vector is in the opposite direction to group velocity, $\mathbf{v}_g\cdot\mathbf{k}<0$, resulting in negative refraction in the second band and correspondingly negative refractive indices.[11]

The bandwidth for obtaining a sharp focus point is a crucial parameter for applications of the left-handed lenses. Due to the resonant nature of the metamaterial the bandwidth

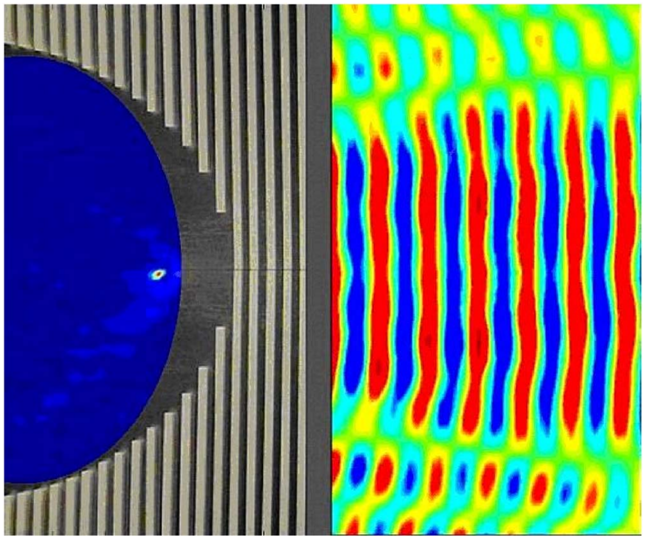

FIG. 4. (Color online) Field maps of the incident source and the emerging plane wave. Scale: on the left side intensity varies from 0 to 0.14, on the right side the real part of S_{21} from −0.03 to 0.02. The source is placed at the focal length 6.15 cm and plane wave is observed at 9.55 GHz.

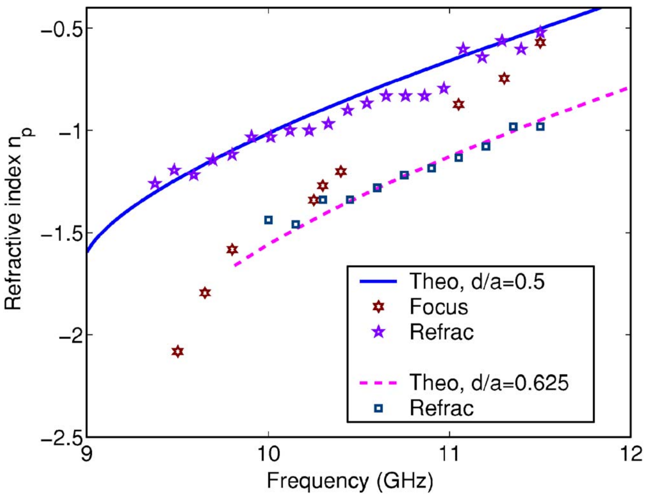

FIG. 5. (Color online) Refractive indices n_p determined from the focusing and refraction experiments for the TM modes of the 1D dielectric PCs with $\epsilon=8.9$. The solid line is for PC1 ($d/a=0.5$) and the dashed line for PC2 ($d/a=0.625$).

is usually restricted to a narrow region and the dispersion is strong.[15] The PC1 reveals a wide bandwidth of 3.8 GHz, which is 35% at the current operating frequencies. The weaker dispersion in the PC makes it a better candidate for focusing a pulse or broadband radiation.

The present PC lens with negative refraction has several advantages when compared to the one with positive refraction. Lenses with reduced geometric aberrations produce sharper image with enhanced resolution and find numerous applications. Larger radius of curvature gives the advantage of reduced aberration in the image formed. A PC lens having the same focal length as that of a conventional lens weighs far less, and is attractive to space applications. The tailor made refractive index achievable in PCs (Ref. 16) allows further control on the focal length and thereby helps to reduce the size of the optical systems.

In conclusion the feasibility of designing a 1D broadband left-handed PC lens is experimentally demonstrated. Negative refraction of plane waves and plano-concave lens focusing is achieved in 1D PCs. The focal length follows the standard laws of geometrical optics combined with negative refraction. The measured values of refractive indices of the lens from both refraction and focusing experiments are in excellent agreement with those determined from band structure calculations. Earlier works have shown that 1D PCs can be used as omnidirectional reflectors.[17,18] The observed negative refraction in 1D PCs reported here, adds diversity to these simple systems.

The authors thank E. Brownell for assistance with the experiments. This work was supported by the Air Force Research Laboratories, Hanscom AFB, and NSF Grant No. PHY-0457002.

[1] V. Veselago, Sov. Phys. Usp. **10**, 509 (1968).
[2] J. B. Pendry, Phys. Rev. Lett. **85**, 3966 (2000).
[3] C. Luo, S. G. Johnson, and J. D. Joannopoulos, Phys. Rev. B **65**, 201104 (2002).
[4] P. V. Parimi, W. T. Lu, P. Vodo, and S. Sridhar, Nature (London) **426**, 404 (2003).
[5] P. Vodo, P. V. Parimi, W. T. Lu, and S. Sridhar, Appl. Phys. Lett. **86**, 201108 (2005).
[6] C. Luo, S. G. Johnson, J. D. Joannopoulos, and J. B. Pendry, Phys. Rev. B **68**, 045115 (2003).
[7] D. Schurig and D. R. Smith, Phys. Rev. E **70**, 065601 (2004).
[8] J. Chen, C. Radu, and A. Puri, Appl. Phys. Lett. **88**, 071119 (2006).
[9] R. A. Shelby, D. R. Smith, and S. Schultz, Science **292**, 77 (2001).
[10] C. G. Parazzoli, R. B. Greegor, K. Li, B. E. C. Koltenbachand, and M. Tanielian, Phys. Rev. Lett. **90**, 107401 (2003).
[11] P. V. Parimi, W. T. Lu, P. Vodo, J. Sokoloff, J. S. Derov, and S. Sridhar, Phys. Rev. Lett. **92**, 127401 (2004).
[12] Z. Lu, J. A. Murakowski, C. A. Schuetz, S. Shi, G. J. Sneider, and D. W. Prather, Phys. Rev. Lett. **95**, 153901 (2005).
[13] J. D. Joannopoulos, R. D. Meade, and J. N. Winn, *Photonic Crystals: Molding the Flow of Light* (Princeton University Press, Princeton, NV, 1995), Chap. 4, p. 38.
[14] R. Kronig and W. G. Penney, Proc. R. Soc. London, Ser. A **130**, 499 (1931).
[15] C. G. Parazzoli, R. B. Greegor, J. A. Nielsen, M. A. Thompson, K. Li, A. M. Vetter, M. H. Tanielian, and D. C. Vier, Appl. Phys. Lett. **84**, 3232 (2004).
[16] P. Vodo, P. V. Parimi, W. T. Lu, S. Sridhar, and R. Wing, Appl. Phys. Lett. **85**, 1858 (2004).
[17] Y. Fink, J. N. Winn, S. Fan, C. Chen, J. Michel, J. D. Joannopoulos, and E. L. Thomas, Science **282**, 1679 (1998).
[18] M. Ibanescu, Y. Fink, S. Fan, E. L. Thomas, and J. D. Joannopoulos, Science **289**, 415 (2000).