

Focusing by planoconcave lens using negative refraction

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(Received 1 February 2005; accepted 7 April 2005; published online 10 May 2005)

We demonstrate focusing of a plane microwave by a planoconcave lens fabricated from a photonic crystal having a negative refractive index and left-handed electromagnetic properties. An inverse experiment, in which a plane wave is produced from a source placed at the focal point of the lens, is also reported. A frequency-dependent negative refractive index, $n(\omega) < 0$ is obtained for the lens from the experimental data which match well with that determined from band structure calculations. © 2005 American Institute of Physics. [DOI: 10.1063/1.1927712]

Negative refraction in left-handed materials (LHM)^{1–6} has triggered intense interest in designing microwave and optical elements, a flat lens being one of them.⁷ In the case of the flat lens, the waves entering from the source refract negatively on both interfaces and meet constructively on the far side of it. Thus, a flat lens applies phase correction to the propagating waves—similar to a conventional lens made of a naturally available material and having a positive index of refraction. However, it operates only when the source is close to the lens.^{7,8} For a majority of applications of lenses in optics, astronomical telescopes, commercial, and defense microwave communications, far-field imaging is required. Negative refraction allows focusing of a far-field radiation by concave rather than convex surfaces,⁹ with the advantage of reduced aberration for the same radius of curvature and changes many commonly accepted aspects of conventional optical systems. Of the two classes of LHM currently being investigated, focusing using a planoconcave lens made of a left-handed metamaterial (MM) fabricated by interleaving arrays of wire strips and split ring resonators was demonstrated experimentally by Parazzoli *et al.*¹⁰

In this letter, we demonstrate that a real image of a far-field radiation can be produced using a left-handed photonic crystal (PhC) lens. We also report an inverse experiment in which the lens produces plane waves from a point source placed at the focal length. The frequency-dependent refractive index, $n(\omega)$, determined from the experimental data, is in complete agreement with that predicted by the theory using band structure calculations. The results confirm that far-field focusing is realizable and opens the door for several applications of the LHM in the far-field region.

Microwave focusing measurements are carried out using three planoconcave lenses made of a dielectric PhC. The radii of curvature of the lenses are 13.5, 17.5, and 22 cm. The two-dimensional PhC consists of a periodic array of alumina rods in air, arranged on a square lattice and having dielectric constant, $\epsilon=8.9$. The ratio of the radius of the alumina rods to the lattice spacing is $r/a=0.175$. Microwave measurements are carried out in a parallel plate waveguide. An X-band waveguide kept at a distance of 150 cm from the flat surface of the lens acts as a microwave source. The emitted wave travels through the parallel-plate waveguide and the eventual plane wave is made to incident on the flat sur-

face of the lens. The propagation of the wave inside the PhC is along ΓX direction of the Brillouin zone. Field maps of the incoming plane wave and the emerging radiation, on the far side, are captured using a monopole sensor on a ground plane. The sensor is hooked up to an automated X-Y translational stage which scans for the electric-field component of the microwaves in the region of interest as shown in Fig. 1. An HP-8510C network analyzer is used for measuring the transmission characteristics.

Figure 2 shows a sharp focus point achieved at 10.1 cm using the planoconcave PhC lens of radius of curvature 13.5 cm. From left to right, the incoming plane wave, a real picture of the PhC lens, and the emerging mapped field are shown. A clear focusing point is observed in the frequency range of 9.265–9.490 GHz. Note that the direction of the energy flow changes only at the second interface of the planoconcave lens. To validate that the focusing is due to negative refraction, an inverse experiment is carried out, in which a point source is kept at the observed focal point of the lens. As can be seen from Fig. 3, a circular wave front from the point source after passing through the lens emerges as a plane wave. These two results validate the behavior of the left-handed planoconcave lens. A similar lens made of a naturally available material, on the contrary, would not be able to produce a real focus point as the present PhC lens does, but rather produce a diverging beam.

The refractive index of the lens is determined using the lens equation $n=1-R/f$, where R is the radius of curvature

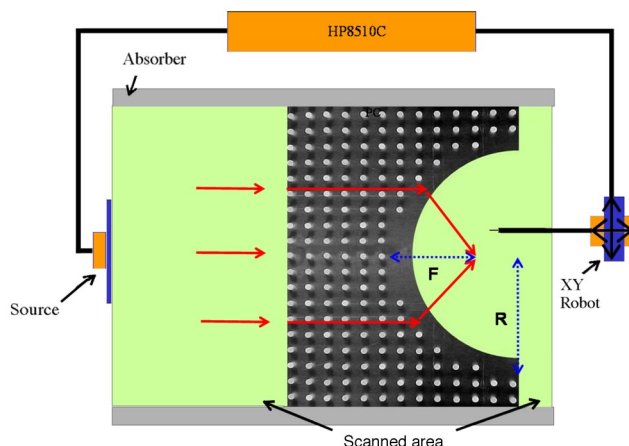


FIG. 1. (Color online) Schematic diagram of the microwave focusing experimental setup.

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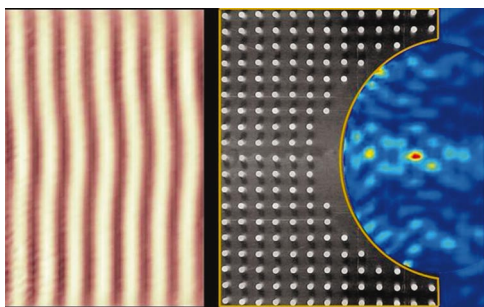


FIG. 2. (Color) Focusing by a planoconcave PhC lens having radius of curvature 13.5 cm. The focus point observed at 9.31 GHz is 10.1 cm from the concave lens surface. A photograph of the PhC is superimposed on two MATLAB surface plots to obtain the final figure. Dark strip in the center is a schematic representation of the area between the lens and the incoming wave. On the left side, field map of the incoming plane wave is shown (real part of transmission coefficient) and on the right side, intensity of the focus point. Scale: On the left, the real part of S21 varies from -0.025 to 0.025 , on the right side intensity from 0 to 1.6×10^{-3} . Dimensions of the figure are 49×34 cm². PhC lattice spacing is 1.8 cm and the packing density of the square lattice is determined from the ratio $r/a=0.175$.

and f is the focal length. Using this description, we get $n = -0.4$ at 9.25 GHz. Note that real focusing by a planoconvex lens is achieved with $n > 1$, $R < 0$, while for the planoconcave lens with $R > 0$, $n < 1$. In Fig. 4, experimentally determined $n(\omega)$ for all three lenses is shown. It can be seen that for $R=13.5$ cm a sharp focus is achieved in the frequency range from 9.25 GHz to 9.5 GHz, for $R=17.5$ cm in the range from 8.5 GHz to 9.2 GHz, and for $R=22$ cm, from 7.8 GHz to 8.1 GHz.

The nature of the left-handed electromagnetism and focusing can be understood from the dispersion characteristics of the PhC. Figure 5 shows band structure calculated for the PhC using a plane wave expansion method. From the band structure, it can be deduced that in the second band for propagation along ΓX direction, the wave vector k is in opposite direction to group velocity, $v_g : v_g \cdot k < 0$,^{11,5} resulting in negative refraction in the second band and correspondingly negative refractive indices. From the band structure the theoretical refractive index $n = ck/\omega$ is determined. The solid line in Fig. 4 shows $n(\omega)$ obtained at various frequencies. As the frequency increases from 7.8 to 9.8 GHz, the theoretical n increases from -1.1 to 0 . Note that both the experimentally obtained refractive indices and the theoretically calculated values are in complete agreement.

In Fig. 4, it can be observed that $n(\omega)$ determined for each radius of curvature fall in certain frequency windows,

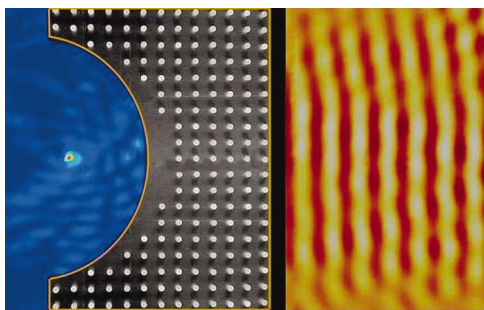


FIG. 3. (Color) Field maps of the incident source and the emerging plane wave. Scale: On the left side intensity varies from 0.005 to 0.055, on the right side the real part of S21 from -0.025 to 0.015 . Dimensions of the figure are 49×34 cm².
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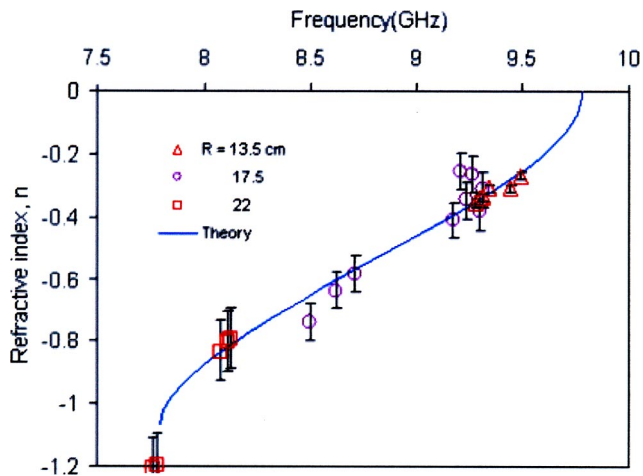


FIG. 4. (Color online) $n(\omega)$ determined from the focusing experiments using PhC lens. Note that the theoretical prediction (solid line) matches excellently with the experimental results.

and these windows move downward as the radius of curvature is increased. The inhomogeneity of the PhC lens leads to corrugation on the concave surface. Windowing of refractive indices for each radius of curvature could be due to corrugation, as the conservation of wave vector on the refracting surface is considerably effected from such corrugation leading to no focus point. However, in the regions where focusing is achieved, corrugation has minimal effect on the sharpness of the image.

The present lens has several advantages when compared to the one with positive index. Lenses with reduced geometric aberrations produce sharper image with enhanced resolution and find numerous applications. For any value of $n < 0$, the radius of curvature of the left-handed PhC lens is always larger than that of its counterpart, a positive index planoconvex lens. Larger radius of curvature gives the advantage of reduced aberration in the image formed. Secondly, a PhC lens having the same focal length as that of a conventional lens weighs far less, and is suitable for space applications. The tailor made refractive index achievable in PhC materials¹² allows further control on the focal length and thereby helps reduce the length of the optical systems.

Bandwidth for obtaining a sharp focus point is a crucial parameter that decides the eventual applications of the left-handed lenses. The present PhC lens reveals a wide bandwidth of 2 GHz, which is 22.7% at the current operating frequencies. In comparison to a planoconcave lens made of

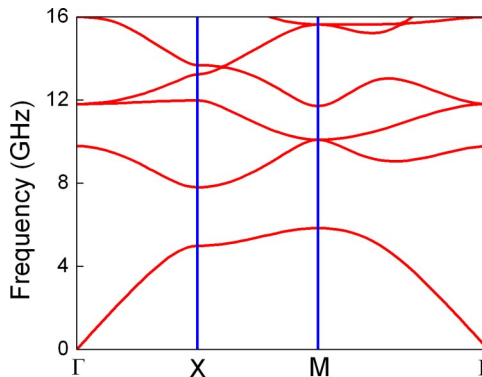


FIG. 5. (Color online) Band structure of the PhC calculated from plane wave expansion method.

the MM, the PhC bandwidth is much larger. Due to the resonant nature of the MM the bandwidth is usually restricted to a narrow region and the dispersion is stronger.¹³ The weaker dispersion in the PhC makes it a better candidate for focusing a pulse or broadband radiation.

In conclusion the feasibility of designing a broadband left-handed lens is experimentally demonstrated. Focusing of plane waves by the planoconcave PhC lens is achieved for three different radii of curvature. The focal length follows the standard laws of geometrical optics applied with negative refraction. Further, the measured values of refractive indices of the lens are in complete agreement with those determined from band structure calculations.

This work was supported by the Air Force Research Laboratories, Hanscom AFB under Contract No. F33615-01-1-1007, and the National Science Foundation.

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