Quantitative analysis of nonlinear microwave surface impedance from non-Lorentzian resonances of high Q resonators

T. Jacobs,[a] Balam A. Willemsen,[b] and S. Sridhar[c]
Physics Department, Northeastern University, Boston, Massachusetts 02115

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A method of analyzing the response of nonlinear high Q resonant circuits, such as superconducting microwave cavities, is presented. The method enables one to extract the nonlinear surface impedance, $Z_s(H_{rf})$, from the complex transmission amplitude, $S_{21}(f)$, as frequency, f, is swept across the resonance. Sensitivity to small changes in the microwave field strength ($\Delta H_{rf}<1$ mOe) can thus be obtained. © *1996 American Institute of Physics.* [S0034-6748(96)03409-0]

The nonlinear response of high T_c superconductors, even at moderate microwave powers, has been cited as a major limitation for microwave applications of the cuprate superconductors.[1,2] There is a need for an accurate extraction of quantities describing the nonlinearities as a prelude to understanding their origins. In this article we discuss a method of extracting one such quantity, the surface impedance as a function of microwave field strength $[Z_s(H_{rf})]$, from the non-Lorentzian resonances of a high Q superconducting microwave cavity.

For linear response, a high Q resonator can be modeled by a simple $R-L-C$ circuit. In the case of weak coupling to the resonator the transmission function is a pure Lorentzian, i.e.,

$$S_{21}=\frac{S_{21\,\max}}{1+iQ\left[1-\left(\frac{f_0}{f}\right)^2\right]}\approx\frac{S_{21\,\max}}{1+i2Q\left[\frac{f}{f_0}-1\right]}, \tag{1}$$

where f_0 is the resonant frequency and Q is the resonance quality factor. The approximation on the right hand side is valid if the analyzed frequency range $\Delta f=(f_0-f)\ll f_0$. For fixed coupling we have $S_{21\,\max}=KQ$, where K is a constant.

The desired quantity, the surface impedance $Z_s=R_s+iX_s$, can be determined from the measured Q and f_0 using the standard expressions

$$R_s=(\Gamma/A)[Q^{-1}-Q_b^{-1}], \tag{2}$$

$$\Delta X_s=(2\Gamma/Af_0)[f_{b0}-f_0], \tag{3}$$

where the subscript b refers to the background values of the same quantities measured without the sample, Γ is a factor given by the cavity geometry and A is the effective surface area of the sample corrected for demagnetization effects.

If we rewrite Eq. (1) as

$$S_{21}^{-1}=\Re(S_{21}^{-1})+i\Im(S_{21}^{-1})=\frac{1}{KQ}+i\,\frac{2}{K}\left[\frac{f}{f_0}-1\right], \tag{4}$$

it becomes obvious that in the case of linear response the real part of $S_{21}^{-1}(f)$ is constant and the imaginary part is a linear function of f. This can be used as a very stringent test of linear response.

In the nonlinear case the effective values of Q and f_0 vary as the frequency is changed. This apparent frequency dependence arises from the fact that the absorbed power in the cavity varies over the resonance, and hence so does the rf field strength.

In order to extract the Q and f_0 for each point in the trace, the coupling constant K has to be eliminated. This can be done by a separate measurement or by extrapolating linear sections of $\Im[S_{21}^{-1}]$ to a common intercept at $f=0$. The high Q and f_0 follow from the real and imaginary parts of S_{21}^{-1}:

$$R_s=\frac{\Gamma}{A}[K\Re(S_{21}^{-1})-Q_b^{-1}], \tag{5}$$

$$\Delta X_s=\frac{2\Gamma}{A}\left\{\frac{f_{b0}}{f}\left[\frac{K}{2}\,\Im(S_{21}^{-1})+1\right]-1\right\}. \tag{6}$$

In order to obtain $Z_s[H_{rf}(f)]$ the field strength corresponding to each f is determined using the formula $H_{rf}=\sqrt{2Q\langle P_{abs}\rangle/\Gamma}$. While Q was already determined above, $\langle P_{abs}\rangle$ is determined by measuring the absolute input power $\langle P_{in}\rangle$, the reflected power $\langle P_{ref}\rangle$, and the transmitted power $\langle P_{trans}\rangle$ using power meters and accounting for the known attenuation in the coaxial cables.

The experimental setup used here is described in Ref. 3. The resonance frequency f_0 is $\approx$10.1 GHz, the analyzed frequency span Δf is $\leqslant$4000 Hz, the empty cavity quality factor is $Q_b\approx5\times10^7$ and the quality factor with typical samples is $Q\approx5\times10^6$. The analysis of the empty cavity resonances shows no evidence of nonlinearities, i.e., the resonances are well described by two parameters which do not depend on H_{rf}. Linear response is also observed for the vast majority of samples at low microwave field strengths in the cavity (usually $H_{rf}<10$ Oe).

Steplike nonlinearities have been observed in the $\hat{c}$-axis response of $Bi_2Sr_2Ca_1Cu_2O_{8+\delta}$ (BSCCO) single crystals (Fig. 1).[4] The analysis of these resonances using the method described here shows that the response can then be broken down into regions of quasilinear response, where the Z_s is only weakly dependent on the microwave field strength. Focusing our attention on R_s alone [Fig. 1(c)] it is clear that abrupt jumps in dissipation occur at characteristic values of

[a]Present address: Forschungszentrum Karlsruhe, IMF I, 76021 Karlsruhe, Germany.
[b]Also at: Rome Laboratory, Hanscom AFB, MA. Present address: Superconductor Technologies, Santa Barbara, CA.
[c]Electronic-mail: srinivas@neu.edu

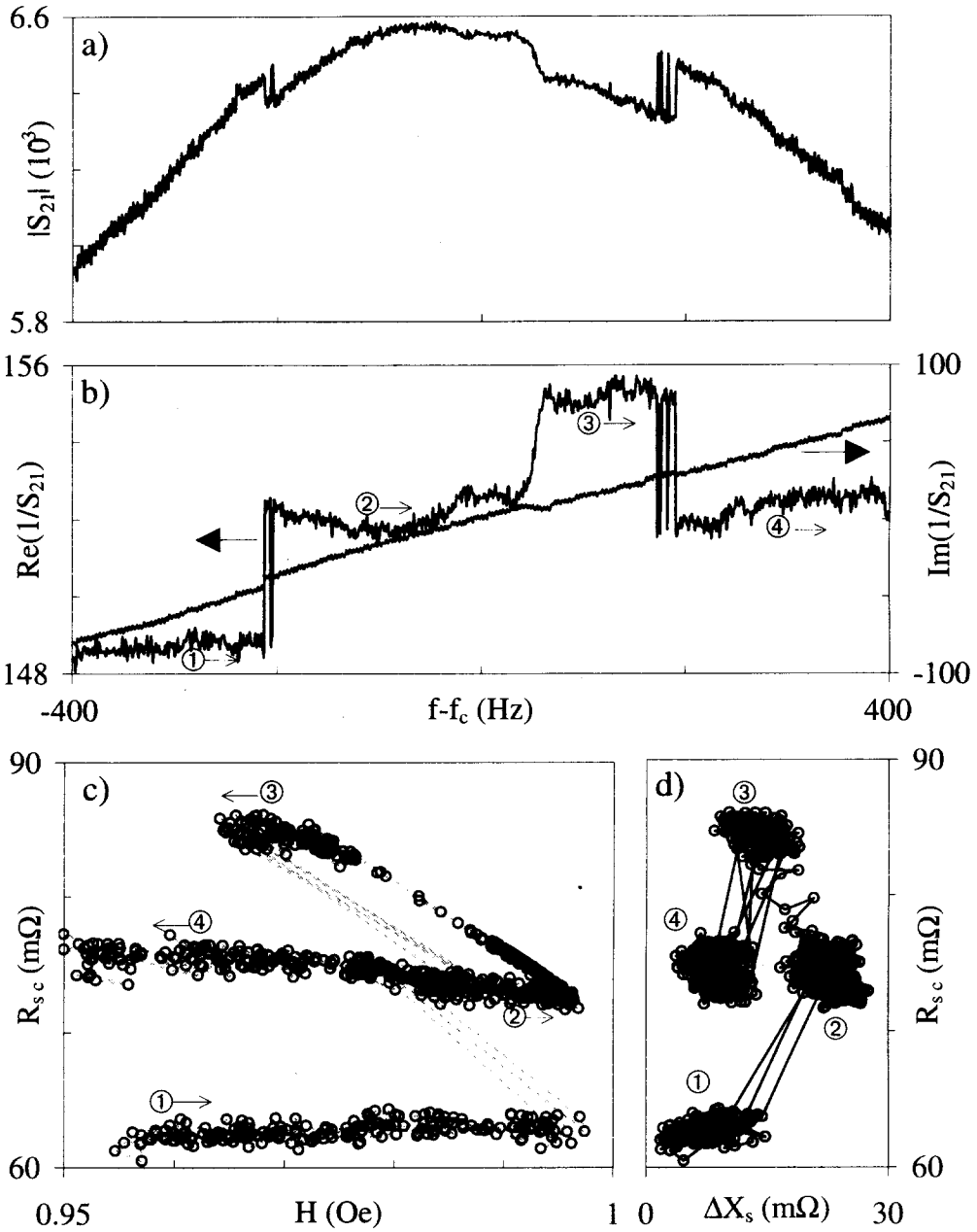

FIG. 1. Sample analysis of a nonlinear resonance of BSCCO at 4 K. a) Transmitted signal $|S_{21}|$ vs f. b) Real and imaginary part of S_{21}. The circled numbers identify different regions of the response. The frequencies axes are referenced to $f_c = 10.102648056$ GHz. c) Extracted $\hat{c}$-axis surface resistance vs H_{rf}. d) Surface resistance vs surface reactance. Each point of the trace corresponds to a different field amplitude.

H_{rf}. The changes in R_s (and hence Q) lead to the changes in H_{rf} observed at the jumps. Interestingly, R_s appears almost quantized. The hysteresis evident in $R_s(H_{rf})$ (where the decreases in R_s occur at different field levels than the increases) is directly related to the observed asymmetry of the resonance. The changes in R_s are accompanied by simultaneous changes in ΔX_s, as can be seen in Fig. 1(d). Further details of the experiments on BSCCO are discussed in Ref. 4. Similar steplike nonlinearities have also been observed in the ab-plane response of twinned $YBa_2Cu_3O_{7-\delta}$ crystals.[5]

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[1] Z.-Y. Shen and C. Wilker, Microwaves & RF, 129 (1994).
[2] P. H. Carr and B. R. McAvoy, IEEE Trans. Microwave Theory Tech. **39**, 1445 (1991).
[3] S. Sridhar and W. L. Kennedy, Rev. Sci. Instrum. **59**, 531 (1988).
[4] T. Jacobs, B. A. Willemsen, S. Sridhar, Q. Li, G. D. Gu, and N. Koshizuka, in *10th Anniversary* HTS *Workshop On Physics, Materials & Applications* (World Scientific, Singapore, 1996).
[5] T. Jacobs, Diploma thesis, Universität Karlsruhe, 1996.