Microwave response of thin-film superconductors

S. Sridhar
department of Physics, California Institute of Technology, Pasadena, California 91125
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The interaction of microwaves with superconductors was explored via extensive measurements
of the surface resistance $R_s$ and reactance $X_s$ at 10 GHz of superconducting Sn. The
measurements were carried out as functions of thickness $d$ and temperature $T$ for Sn films
ranging in thickness from 190 Å to bulk. By varying the thickness with accompanied variation
of mean free path, we were able to examine the wave-vector dependence of the microwave-
superconductor interaction. An analysis, based on the Bardeen-Cooper-Schrieffer
electrodynamical kernel, was developed for calculating $R_s$ and $X_s$ as functions of $d$, $T$, and
frequency. Details of the measurement techniques and analysis are presented. Experiment and
theory agree excellently, with no adjustable parameters. For films with $d < 800$ Å we find that
the electrodynamics is local because of impurity scattering, while for thicker films nonlocal
effects are important. The results imply that superconducting films with microwave response
that agrees with theoretical expectations can be fabricated for potential applications in a
variety of superconducting structures.

I. INTRODUCTION

The interaction of electromagnetic waves with superconductors is of importance both from fundamental and
technological viewpoints. Early studies of microwave absorption by superconductors played an important role in elu-
cidating the physics of superconductivity, and subsequently, a variety of applications attempted to exploit the low-loss property of superconducting materials at high frequencies.

Consequently, there have been many investigations of the microwave properties of superconductors. These previous investigations have been primarily concerned with two extreme limits of thickness of the superconductor—very thin films, where there is substantial transmission of microwaves through the films, and bulk superconductors. Recently, both the questions of physics and the demands of technology seem to call for a more detailed understanding of the microwave properties of superconducting films. Areas of recent fundamental interest are nonequilibrium superconductivity\(^1\) and the interplay of localization and superconductivity.\(^2\) Examples of proposed applications\(^3\) are superconductor-insulator-superconductor (SIS) mixers, low-loss microwave filters, high-power microwave switches, and coated sapphire resonators for use as ultrastable clocks and gravity wave detectors.

This paper presents the results of a detailed experimental and theoretical investigation of the interaction of microwaves with superconducting films. By varying temperature and film thickness, the temperature and wave-vector dependence of this interaction was explored. The extensive experimental results were then compared with the predictions of the microscopic BCS theory.

In this paper results are reported of extensive measurements of surface resistance $R_s$ and surface reactance $X_s$ (surface impedance $Z_s = R_s + iX_s$) over a wide range of temperatures $T$ and thickness $d$ of superconducting Sn films. The thickness $d$ was varied from $d = 200$ Å to bulk ($d = 2.4$ μm)—the corresponding change in $R_s$ being almost a factor of 10. This thickness range brackets the fundamental length scales of the superconductor involved since the superconducting penetration depth $\lambda_s = 510$ Å and bulk coherence length $\xi_0 = 2300$ Å. Measurements are also reported for $1-5$ μm Sn films plated on Cu and for a 1/2-in.-thick Sn disk.

A numerical program was also developed for computing the surface impedance of thin-film superconductors. The usual calculation of $Z_s$ for bulk superconductors was adapted to the case of thin films by incorporating the electrodynamical boundary conditions due to the presence of both surfaces and assuming specular reflection for electrons at both the interfaces. The Khalatnikov-Abrikosov\(^4\) formulation of the kernel $K$ was used in its general form. The calculation is expected to be applicable over a wide range of thicknesses and material parameters.

The results for $R_s(T,d)$ and $X_s(T,d)$ were compared with the numerical calculations for all the temperatures and thicknesses studied. Only measured quantities, the transition temperature $T_c$, frequency $\omega/2\pi$, and the mean free path $l$ determined from the normal-state resistivity, were used in the comparison. There were no adjustable parameters. The agreement between experiment and theory is excellent and holds over orders of magnitude variation in the impedance.

The conclusions of this work are of importance in several ways. First, the remarkable agreement of theory and experiment for several orders of magnitude variation of $R_s$ and $X_s$, and for the electronic mean free path, is clear evidence that even for very thin films the electromagnetic response can be well characterized in terms of the microscopic BCS theory. The results of this work may also be viewed as one of the more stringent tests of the BCS theory. Further, the material fabrication and measurement techniques, together with the numerical calculations described, may be used as
was a provision for condensing liquid $^4$He inside. The apparatus was immersed in liquid $^4$He in a stainless-steel Dewar and shielded by a mu-metal can (residual magnetic field $\approx 10^{-3}$ G). Temperature was varied by pumping on the $^4$He bath and could be regulated to within 5 mK.

All measurements were done by using the TE$_{011}$ mode of the resonator. This mode was chosen as it has no electric fields normal to the surface, thus minimizing spurious absorption due to dielectric surface layers and also possible electron loading. Further, there are no currents traversing the In joint connecting the two cavity sections. The dimensions of the cavity were chosen to produce maximum magnetic fields in the end plate on a ring of radius 0.411 in.

It is essential to lift the degeneracy between the TE$_{011}$ and the TM$_{zz}$ modes in order to maintain the advantages listed above. The central pumping hole decreases the frequency of both the TM modes. The lack of azimuthal symmetry, because of the coupling holes, further lifts the degeneracy of the two TM modes.

The final mode configuration for the resonator is depicted in Fig. 1. From an examination of the $R_s$ of a fully Pb-plated cavity, the cross coupling between the modes was shown to contribute no more than 1% to the TE$_{011}$ absorption. (This was deduced from the fact that in going through the In superconducting transition at 3.41 K, there was no measurable effect on $R_s$ for the TE$_{011}$ mode.) Thus the contribution of the In joint to the measurements was completely negligible. Further, the contribution of the Pb surface to the absorption is <1% at all temperatures.

The surface resistance $R_s$ of the material constituting the walls of the cavity can be determined by measuring the unloaded resonator $Q$ and employing the relation

$$R_s = \frac{\Gamma}{Q}. \tag{1}$$

$\Gamma$ is a geometrical factor and is given by $\Gamma = \omega \rho (\varepsilon_0 E_o^2 + \mu_0 B_o^2) dV/2J B_o^2 dS$, where $\varepsilon_0$ and $\mu_0$ are, respectively, the permittivity and permeability of free space (since the resonator is evacuated), $E_o$ and $B_o$ are, respectively, the microwave electric and magnetic fields, and $V$ and $S$ refer to volume and surface integrations, respectively. $\Gamma$ is different between the two cases of a completely homogeneous resonator $T_s$ and the case where only the bottom section contributes to the absorption $\Gamma_s$. The latter case applies, since the bottom plate is covered with a low $T_s$ material (Sn), and the remainder is Pb plated. The calculated geometrical factors for the two cases for the TE$_{011}$ mode are $\Gamma_{N011} = 709$ (\Omega/\square), $\Gamma_{P011} = 2990$ (\Omega/\square), and their ratio $\Gamma_{P011}/\Gamma_{N011} = 4.22$.

Equation (1) was used to determine the surface resistance by a measurement of the resonator $Q$. Pulsed microwave power was fed through one probe, and the cavity field amplitude was picked up with the other and detected (Fig. 2). The detection was carried out by heterodyning to an intermediate frequency of $\sim 25$ MHz and the pulse shape observed in real time on a fast scope. The pulse shape exhibited exponential decay because of the losses. The time constant $\tau_s$ of the cavity amplitude envelope was measured from the unloaded $Q = \pi f_s$. Weak coupling was ensured by verifying that the $Q$ was independent of the probe positions. A typical
III. SAMPLE PREPARATION

Three types of superconducting configurations were studied: (a) thin films evaporated on sapphire, glass, or OFHC Cu, (b) thick (2–5 μm) films electroplated on OFHC Cu, and (c) bulk 2.25-in.-diam × 1-in.-thick Sn disk. The substrates for types (a) and (b) were all 2.25 in. diam × 1 in. thick.

Thin films of Sn were evaporated from Mo boats at a pressure of 5 × 10⁻⁶ Torr. The substrates were cooled to 77 K by means of liquid N₂. Optically reflecting mirror-perfect films were obtained (with polished sapphire substrates) which had no visible structure on a resolution scale of 500 Å upon examination with a scanning electron microscope at a magnification of 20 000.

Samples of type (b) were obtained by electroplating Sn onto polished disks of OFHC Cu in a Sn(BF₄)₂ bath to which HBF₄, β-naphthol, and gelatin were added (the recipe is described in Ref. 8). The plating current density was typically 5 × 10⁻² Å/sec. The resultant coating was optically bright in appearance, but had surface structure approximately 2 μm in size. The specimen was rinsed with water and acetone and preserved in vacuum before assembly.

The Sn disk was made by melting and casting 99.99% Sn sheet into a disk. The disk was machined and polished with 1–3 μm levigated alumina. The surface had clearly observable grain size of approximately a few millimeters.

IV. THEORY OF THE SURFACE IMPEDANCE OF THIN FILMS

The derivation of the surface impedance of bulk metals is based upon two sets of equations. One is the Maxwell equations with appropriate interface boundary conditions, which specifies the electrodynamics. The other is the material or constitutive equation (the analog of Ohm’s law), which gives the relation between current and applied field.

For a superconductor the constitutive equation is a generalization of London’s equation relating current J and vector potential A, to include wave-vector q and frequency ω dependence:

\[ J(q, ω, T) = (-1/μ₀)K(q, ω, T)A(q, ω), \tag{2} \]

where K is the so-called kernel, and is temperature dependent and complex. In general, K also involves parameters specific to the material.

For specular reflection the problem of a plane wave incident on a semi-infinite metal is solved by replacing the metal by a current sheet at the interface, the magnitude of the current being determined by the B field at the surface. Use of this artifice leads to the following expression for the surface impedance for a bulk metal:

\[ Z_s = \frac{jμ₀q}{π} \int_{q_1}^{q_2} \frac{dq}{q + K(ω, T, q)}. \tag{3} \]

To derive the surface impedance for a corresponding thin film we assume that only the boundary conditions are modified and that the second interface influences the electrodynamics. The assumption here is that the constitutive equation (2) is the same as that for bulk, provided the appropriate thin-film material parameters like mean free path l are used.
In addition, it is necessary to specify the nature of the reflection of the electrons at the interfaces. We consider only the case of specular reflection at the film surfaces as the expressions for diffuse reflection are quite intractable.

For a thin-film filling the space $0 < x < d$ with the electromagnetic field incident on one side of the film at $x = 0$, the film can be replaced by an infinite set of current sheets spaced $2d$ apart covering the entire space (Fig. 3). We assume that there is no transmission through the film so that $B(x = d) = 0$. This assumption is true provided that $2\pi\lambda^2/dv < 1$, where $\lambda$ is the bulk penetration depth (or skin depth in the case of a normal metal), and $v$ is the free-space wavelength. If this condition is satisfied, then $B(d) \approx (2\pi\lambda^2/dv)B(0)$ and is negligible. Thus the magnetic field decays (linearly for a very thin film) to zero across the film. It is perhaps worthwhile to emphasize that, counter to intuition, even films with $d \approx \lambda$ have negligible transmission.

Within the above transformation, $Z_1(\omega, T, d)$ can be calculated in a manner exactly analogous to the bulk case. The result is

$$Z_1(\omega, T, d) = \frac{i\mu(\omega)}{d} \sum_{n} \frac{1}{q_n^2 + K(\omega, T, q_n)}, \tag{4}$$

where $q_n = n\pi/d$. Once the kernel $K$ is specified, $Z_1$ can be calculated.

It is useful to illustrate the above consideration for the simple case of local\(^4\) electrodynamics (London limit) at $T = 0$. Then $K(q) = K(\infty) = 1/\lambda_L^2$, where $\lambda_L$ is the London penetration depth. Then $Z = i\mu(\omega)\lambda_L \coth(d/\lambda_L)$ and is purely reactive, since there is no absorption. The spatial dependence of the field-induced supercurrent may also be simply expressed. For the magnetic field $B(z) = B(0) \sinh((d-z)/(\lambda_L)) / \sinh(d/\lambda_L)$, where $B(0)$ is the magnitude of the incident field. For the current $J(z) = J(0) \cosh((d/\lambda_L)(1-Z/d))/\sinh(d/\lambda_L)$.

Figure 3 displays the current and field profiles calculated for various values of $d/\lambda_L$.

In bulk both the fields and currents decay exponentially in the metal. As the thickness is decreased, the current density is greater, because the current is squeezed into a narrower region, although the total current is determined by the applied field and has to remain fixed. Consequently, the impedance increases, for purely electrodynamic reasons. In a very thin film the current becomes almost uniform, and the field decays linearly to zero across the film, as shown in Fig. 3. Essentially, the same considerations apply for the normal state also.

Another simple case that can be considered is the local limit at finite temperatures. Here it is convenient to express the constitutive relation in terms of the Mattis-Bardeen\(^10\) complex conductivity $\sigma_1 = \sigma - i\sigma_2$; $J = \sigma E$. Detailed expressions for the $\omega$ and $T$ dependencies of the $\sigma_1$ and $\sigma_2$ normalized to the normal state conductivity $\sigma_n$ are given by Mattis and Bardeen\(^10\). In the above form the complex impedance can be calculated by using Eq. (4):

$$Z_1(\omega, T, d) = R_n(2\pi/\alpha) \coth(d/\lambda_n), \tag{5}$$

where $\alpha = (2\pi(\sigma_n/\sigma))^1/2$, $R_n = \sigma_n/\lambda_n$ is the normal-state bulk surface resistance, and $\lambda_n$ is the classical skin depth. Equation (5) is a useful, simple form for calculating $Z_1$ for thin films. However, as discussed later, it does not work very well in thick films where nonlocal effects are important. In the limit $d \to 0$, Eq. (5) reduces to the intuitive result $Z_1 = 1/d\alpha$.

The above limits are not applicable in general, and one needs a more complete specification of the kernel. Khalatnikov and Abrikosov\(^4\) have given an extensive theory of the interaction of electromagnetic radiation with superconductors in terms of the BCS theory and have also obtained detailed expressions for the kernel $K$ as a function of wave vector, frequency, temperature, and parameters specific to the material. These expressions were used by Halbritter\(^11,12\) to numerically compute the surface impedance of bulk superconductors.

We have employed Halbritter's program to compute numerically $K(\omega, q_n)$ for $q_n = n\pi/d$, where $n = 0, 1, 2, \ldots$. The results were then used in Eq. (4) to calculate $Z_1(\omega, T, d)$. The expression in Eq. (4) converges fairly rapidly, typically for $n < 20$.

The inputs to the computation of $Z_1(\omega, T, d)$ are $T$, $I$, $d$, $\Delta(0)/K\lambda$, $\lambda_L(0)$, and $\xi_0$, where $\Delta(0)/K\lambda$ is the gap ratio ($\approx 1.74$ for weak coupling superconductors), $\lambda_L(0)$ is the London penetration depth at $T = 0$, and $\xi_0$ is the coherence length. Of these the calculation is fairly insensitive to the material parameters $\Delta(0)/K\lambda$, $\lambda_L(0)$, and $\xi_0$. The bulk values for a superconductor reported in the literature\(^5\) were used for these parameters. The measurement of the important parameters is described in the previous section. In this experiment $\omega = 2\pi \times 10^{10}$ rad/s.

V. MEASUREMENT OF SAMPLE PARAMETERS
A. Normal state resistivity and mean free path

After the microwave measurements were completed, the dc resistivity was measured by cutting a 1-cm $\times$ 1-mm...
area with banks for contacts and employing a four-terminal method.

The normal-state resistivity was also determined from the surface resistance $R_s$ in the normal state above $T_c$, for which

$$Z_n(\omega, d) = \sqrt{2i} R_s \coth(\sqrt{2i}d/\delta),$$

where $R_s = (\mu_0 \rho_n/2)^{1/2}$ and $\delta = (2\rho_n/\mu_0 \omega)^{1/2}$. For very thin films, where $d < \delta$, $\rho_n = R_s d$. For larger thickness, Eq. (6) was solved numerically. The deduced resistivities are listed in Table I.

For samples of type (b) and (c) it was not feasible to measure the dc resistivity, and $\rho_n$ was inferred from $R_s$. However, in these cases, the anomalous skin effect comes into play. For $\beta > 2.1$ Chambers$^{13}$ gives the following extrapolated expression:

$$R_n(\omega, d, l \rightarrow \infty) = R_s \times \left[1 + 1.157\beta^{-0.275}\right],$$

where $R_s = R_n(\omega, \infty, \infty) = [\sqrt{3\pi/(\rho_n l)}(\mu_0 \omega/4\pi)^{2}]^{1/2}$ and $\beta = [\rho_n(\mu_0/2)^2/\rho_n^2]$. For $\omega = 2\pi \times 10^{12}$ and by using the free-electron value of $\rho_n(l$ see later), we get

$$\beta = \left[0.864 \left(\frac{R_s}{6.6 \times 10^{-3}} - 1\right)\right]^{-1/3},$$

$$\rho_n = \frac{0.285}{\alpha^{1/3}} \mu\Omega \text{ cm},$$

(7)

where $R_s$ is in $\Omega$. Thus $\rho_n$ at 10 GHz can be determined from $R_s$, and the values are listed in Table I. Since $\alpha = 1.2$ corresponds to $R_s = 1.2 \times 10^{-3} \Omega cm$, these expressions are appropriate to the samples P-2 and B-1 for which $R_s < 1 \times 10^{-3} \Omega cm$, listed in the table.

From the resistivity the mean free path was deduced from the free-electron expression$^{14}$:

$$l = (0.06\rho_n) \mu\text{m},$$

(8)

where $\rho_n$ is in $\mu\Omega \text{ cm}$. The resulting values deduced from the microwave measurements are given in Table I. It will be noticed that $l$ increases with $d$ saturating at $l = 2100 \text{ Å}$ at large thickness. For the very thin films boundary scattering is clearly important, while for the thicker films grain-boundary scattering probably limits $l$.

The resistivity deduced from the dc ($\rho_{dc}$) and microwave measurements ($\rho_{ac}$) were in agreement with the experimental accuracy for all the films except for the thinnest films with $d < 440 \text{ Å}$. For the latter $\rho_{ac} < \rho_{dc}$, deduced from $R_s$. The implications of this are discussed later. The important parameter that enters into the calculation of the superconducting properties is the mean free path, and as discussed later, we find that the appropriate value is that deduced from the microwave measurements.

**B. Transition temperature $T_c$.**

This was determined by both dc and microwave measurements as the temperature at which the dc resistance or microwave surface resistance changed by 5% from the normal-state value. Both measurements agreed to within 0.01 K.

The results of $T_c$ for the different samples are given in Table I. It will be observed that $T_c$ increases with decreasing $l$. One reason that has been advanced$^{15}$ for this is that the electron-phonon interaction gets stronger as $l$ decreases; hence $T_c$ increases.

**VI. COMPARISON OF EXPERIMENTS WITH THEORY**

Samples prepared as described earlier were assembled into the cavity. A measurement of the normal-state surface resistance $R_s$ gave a measurement of the mean free path. The resonator was cooled below $T_c$, and the surface resistance $R_s$ was determined from the resonator $Q$.

The measured parameters $T_c$, $l$, and $\omega$ were then used in the calculation of the surface resistance (Sec. V). For Sn the established values, $\Delta /kT_c = 1.74$, $\lambda = 0 = 340 \text{ Å}$, $\delta_0 = 2300 \text{ Å}$, from the literature$^{5,16}$ were used.

Figure 5 is a plot of $R_s(t, d)$ vs $t = T / T_c$ for some of the samples listed in Table I. The bulk experimental results for $d > 3600 \text{ Å}$ for all configurations (a), (b), and (c) are identical to within experimental accuracy. The theoretical calculations also yield almost a single curve. This is because the surface resistance is insensitive to thickness and mean free path (m.f.p.) for large $d$.
One may ask whether these results are clear evidence that the electrons are specularly reflected at the interfaces. A quantitative calculation for the case of diffuse reflection is unfortunately difficult, since the artifice of Fig. 3 cannot be used. Instead, one has to solve a complicated integrodifferential equation. In bulk exact results can be obtained, the calculated difference between the cases lies within the typical experimental errors. For thin films in the normal state the dc resistivity is calculated to be appreciably different for the two cases only for \( \rho \approx d \). In these experiments \( \rho < d \) experimentally, and hence in the normal state there is not an appreciable difference between the two cases. We expect the same to hold true for the superconducting case also, i.e., by using the actual mean free path \( l \) (which is \( \lesssim d \)), the results for specular and diffuse reflection of the electrons at the interface should not be very different.

It was found that the m.f.p. \( l \) determined from the microwave resistivity was longer (by a factor \( < 2 \)) than the dc m.f.p. \( l_{dc} \) for \( d < 650 \, \AA \). For the thinner films Eq. (6) assumes a frequency-independent \( R_a \). In bulk \( R_a \) has a frequency dependence due to the skin depth, the conductivity being frequency independent well below the plasma frequency. The thinner films are likely to be inhomogeneous and to be characterized by an island structure with highly resistive paths joining the islands. Since the dc current is forced to traverse the highly resistive paths, the measured dc resistance is anomalously higher. On the other hand, the microwave reactive displacement currents across the boundaries of the islands can short out the spurious highly resistive paths. The microwave resistivity correctly describes the scattering in the metal, and the corresponding microwave mean free path \( l \) is the appropriate one to use for the superconducting state. Thus an entirely consistent description of the superconducting surface resistance is possible by using the microwave measurements alone, and this description agrees very well with the experimental results.

**VII. MEASUREMENT OF SURFACE REACTANCE \( X_s(T, d) \)**

The usual method of measuring reactance is to measure the change in resonant frequency of the resonator as the temperature is lowered. This only yields the temperature-dependent changes in the reactance and not its absolute value. However, in the configuration employed in this experiment the shift in resonant frequency was dominated by a pressure-dependent volume change and hence inapplicable to measuring \( X_s \). Further, the above method gives only the relative and not an absolute evaluation.

Consequently, a new technique was developed which involves measuring the emission from the resonator. Emission comes about because of transmission through the film and plate. By using the normal state as a normalization this method enables the absolute measurement of the reactance. It is restricted to films of not too great thickness \( d < 1000 \, \AA \) in this experiment). Before the procedure is described an estimate of the transmitted power is presented to show that the contribution to the \( Q \) due to this loss mechanism is negligible compared to the absorption in the film.

Now \( P_{abs} = |R_s B(0)|^2 \) and \( P_{rms} = |Z_0 B(d)|^2 \), where \( Z_0 \) is the impedance of the substrate. Therefore,
\[ \eta = \frac{P_{\text{min}}}{P_{\text{abs}}} = \frac{Z_0 |B(d)|^2}{R, |B(0)|^2} = \frac{Z_0 Q |B(d)|^2}{2970 |B(0)|}. \]

As discussed in Sec. III, \( B(d)/B(0) \approx 2 \pi d^2/\lambda^2 \), where \( d \geq 0.3 \text{ cm}, \lambda \approx 1000 \text{ A}, \gamma = 3 \text{ cm} \), and has been assumed for the worst case. By using a large value for the \( Q = 10^7, \eta = P_{\text{min}}/P_{\text{abs}} \leq 6 \times 10^{-3}. \)

This radiation loss is clearly an insignificant contribution to the surface resistance in this experiment; however, it can be measured directly. An interesting conclusion that arises from this analysis is that there is an upper limit to the \( Q \) because of the radiation loss in a resonator, which has a thin film constituting some portion of its walls.

The relation between impedance and fields is \( R + jX = E(0)/B(0) \) and \( Z_0 = E(d)/B(d). \) Thus

\[ P_{\text{min}} = \frac{1}{2} \frac{R^2 + X^2}{Z_0} \left( \frac{E(d)}{E(0)} \right)^2 \left( \frac{B(0)}{B(0)} \right)^2. \]

For thin films \( E(d)/E(0) = \cosh(d/\lambda) \). This correction becomes significant for \( d > \lambda \). Hence in the normal state \( P_{\text{min}} = R, \lambda = 0 \). For the superconducting state \( \lambda \),

\[ P_{\text{min}} = \frac{1}{2} \frac{X^2}{Z_0} \left( \frac{\cosh(d/\lambda)}{\lambda} \right)^2. \]

Hence, except very near \( T_c \),

\[ X_t = R_s \left( \frac{P_{\text{min}}}{P_{\text{min}}} \right)^{1/2} \cosh(d/\lambda). \] (8)

\( P_{\text{min}} \) was measured by pressing an \( X \)-band waveguide to the back of the sapphire and detecting the transmitted power. By starting from \( T > T_c, P_{\text{min}} \) was measured for all \( T \) relative to the normal-state value \( P_{\text{min}}, \lambda \) was calculated for the dirty superconductor from \( \lambda = \lambda_p (1 + \xi_0/\lambda)^{1/2} \). By using Eq. (8) \( X_t \) was determined from the measured values of the transmission ratio \( (P_{\text{min}}/P_{\text{min}})^{1/2} \).

The results for \( X_t(t,d) \) for some of the films are displayed in Fig. 6. These measurements are less accurate than those of \( R_s \), as they involve the measurements of very small microwave powers. It is estimated that relative error between values at different temperatures is \( \sim 25\% \), and the error in the absolute magnitude is \( \sim 40\% \). Also shown in Fig. 6 are the results of numerically calculating \( X_t(t,d) \) from the BCS theory, as described in Sec. III. The agreement between theory and experiment is consistent with the experimental accuracy.

VIII. CONCLUSIONS

Extensive measurements were carried out of the surface resistance and reactance of thin superconducting films of Sn and bulk Sn. The measurements yield detailed dependencies of the transport parameters on the electronic scattering mean free path, thickness, and temperature. A detailed numerical program was also developed for calculating the surface impedance of thin films on the basis of the BCS theory. Experiment and theory were compared with no adjustable parameters in the comparison. The agreement is excellent for orders-of-magnitude variation in the surface impedance.

These results imply that superconducting films and thin film structures can be fabricated whose electromagnetic properties at high frequencies are in agreement with theoretical expectations. The measurement techniques and analysis presented here may be used as guidelines for similar material studies.

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