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Tunneling proximity resonances: interplay between symmetry and dissipation

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Abstract

We report the first observation of bound-state proximity resonances in coupled dielectric resonators. The proximity resonances arise from the combined action of symmetry and dissipation. We argue that the large ratio between the widths is a distinctive signature of the multidimensional nature of the system. Our experiments shed light on the properties of 2D tunneling in the presence of a dissipative environment. © 2000 Published by Elsevier Science B.V. All rights reserved.

Tunneling and dissipation are ubiquitous phenomena in physics. A detailed understanding of their combined action would be highly desirable given the relevance of the problem for atomic physics, condensed matter physics, chemistry and biology [1]. However, the incorporation of dissipative effects is by no means trivial. Due to limitations of the available analytical and computational methods, up-to-date descriptions are still restricted to a few manageable cases, the prototype situation involving a bistable potential in 1D [2].

In this Letter we report the observation of novel aspects of tunneling in 2D potentials and its interplay with classical dissipation. In experiments utilizing microwave dielectric resonators, we find that sym-

metry not only plays a crucial role while shaping the eigenstates of the system, but also influences the way they couple to the external environment acquiring a finite width. In the observed resonance multiplets, we find that one of the members is extremely sharp due to the symmetry of the configuration. The large ratios of the observed widths appear to be a peculiar consequence of the multidimensional nature of the system.

The experiments were carried out using MgTi dielectric cylinders placed between two parallel copper plates, 30 cm square, separated by a gap $l = 6.38$ mm (Fig. 1). The disks had diameter $d = 12.65$ mm and dielectric constant $\epsilon_r = 16$. After establishing input/output coupling to the *near* field of the resonators by inserting coax lines terminated by loops, measurements of the transmission amplitude as a function of the frequency were performed using an HP8510B network analyzer.

The eigenvalue problem of a single dielectric resonator can be solved analytically by regarding the

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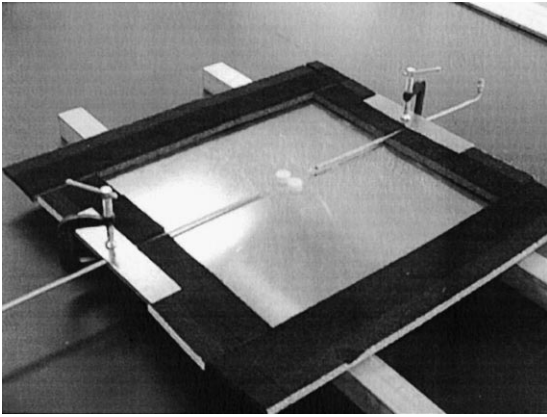


Fig. 1. View of the experimental setup. Shown in the photo are two cylindrical dielectric resonators, and input and output antennas.

system as a waveguide along the direction z orthogonal to the plates [3]. The entire field configuration can be derived from the knowledge of the longitudinal components $\{H_z, E_z\}$ alone, that separately obey the Helmholtz equation:

$$(\nabla^2 + k^2)\{H_z(\mathbf{r}), E_z(\mathbf{r})\} = 0. \quad (1)$$

Here, $\mathbf{r} = (\phi, \rho, z)$ in cylindrical coordinates and $k = \sqrt{\epsilon_r}(\omega/c)$ denotes the medium wave number for a mode at frequency $\omega = 2\pi f$ ($\epsilon_r = 1$ outside the dielectric). For perfectly conducting walls, boundary conditions require $k_z = p\pi/l$, p integer. We have verified through explicit measurements of the field profile that $p \geq 1$, and all modes are evanescent with a decay constant close to the expected value $\kappa_r = \sqrt{k_z^2 - \omega^2/c^2}$ [4]. A generic mode of the dielectric is classified according to its azimuthal, radial and vertical quantum numbers (m, n, p) . If $m = 0$, the mode has cylindrical symmetry and can be either TE or TM. Hybrid HEM modes arise whenever $m > 0$. There is no TEM mode for a dielectric guide. The agreement between the calculated resonances and the data is found to be within 2% for all the peaks [4].

The quality factors Q of the resonances are determined by the observed widths γ in the frequency domain, $Q = f/\gamma$. For a single resonator, estimates of Q are possible by calculating the ratio between

the energy stored per unit time and the average power dissipated [3]. Owing to the *localized* nature of the field eigenmodes, losses due to the open boundary conditions at the edge of the plates are irrelevant, power dissipation being introduced by dielectric and conductor losses. Detailed calculations, which yield results consistent with the measurements, indicate that finite absorption in the metallic plates outweighs dielectric losses by at least a factor 5, thus providing the leading dissipation mechanism [4]. In an equivalent time-domain picture, this implies that the metal acts as an environmental decay channel for the bound electromagnetic modes, the coupling between the dielectric and the metal being proportional to the copper surface resistance.

When two dielectric resonators are placed *in proximity* to each other, each resonance splits into two. The doublets have the structure of a *broad resonance at a lower frequency* f_l accompanied by a *narrow resonance at higher frequency* f_h . This effect is particularly pronounced for TM modes ($H_z = 0$). The most noteworthy example is the TM_{011} single-disk resonance found at 9.45 GHz with $Q_0 \approx 70$, which splits into two peaks with $Q_h \approx 2400$ and $Q_l \approx 50$ for an edge distance $d = 1.0$ mm (Fig. 2). The narrow peak can be experimentally assigned to an *antisymmetric* E_z -field configuration by establishing electric field coupling with the pick-up antenna and by probing the behavior at the mirror symmetry plane [4]. The doublet splitting as a function of the disk separation $\Delta f(d) = f_h - f_l$ is displayed in Fig. 3(a). The splitting vanishes exponentially with d until the limit of noninteracting resonators is approached. The measured decay constant is in good agreement with the single-disk value $\kappa_r = 0.45$ mm⁻¹, as expected on the basis of semiclassical estimates in a tunneling regime where $\kappa_r d \geq 1$ [5]. The widths γ_l, γ_h and their ratio γ_l/γ_h are plotted in Fig. 3(b) and in Fig. 4 (dots) respectively. Again, single-disk behavior is recovered for sufficiently large d , where $\gamma_l, \gamma_h \rightarrow \gamma_0$. For small separations, the width γ_h is highly suppressed, leading to the high Q -values noted above. As indicated by Fig. 4, a maximum ratio $\gamma_l/\gamma_h \approx 50$ is seen at $d = 0.73$ mm. It is very remarkable that, thanks to the proximity effect, Q 's in the range of 10^3 are achievable without resorting to closed-walls cavities.

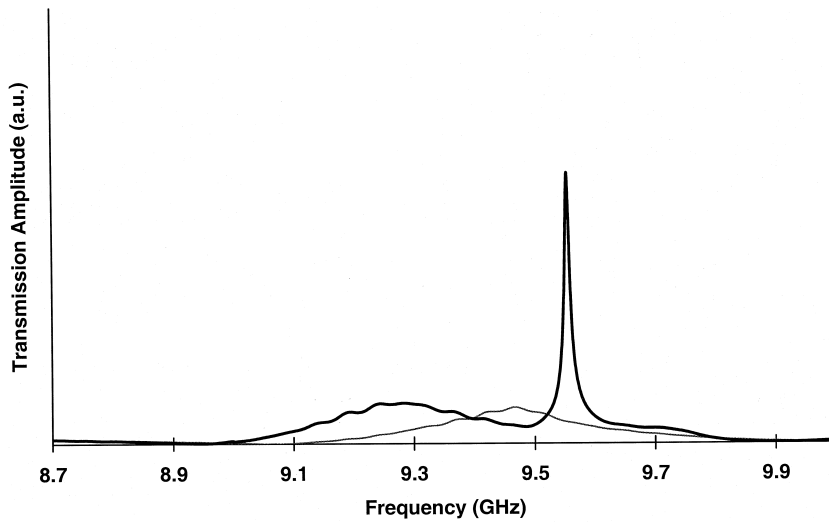


Fig. 2. Experimental proximity resonances for two coupled resonators operating in the TM_{011} mode at 9.45 GHz.

A quantitative account of the above results can be only achieved by numerically solving Eq. (1) with the appropriate boundary conditions. Even if the problem is simplified since the z -dependence is separable, an accurate calculation of the lineshape factors Q requires the complete knowledge of the electric and magnetic field distribution within the cavity

volume. By referring to [4] for more detail on the full electromagnetic analysis, our primary goal here is to gain simple qualitative insights. Let us focus henceforth on the TM_{011} configuration. In the two-disk system, the splitting into modes of well defined parity is easily understood as a consequence of the perturbation introduced by the resonator–resonator

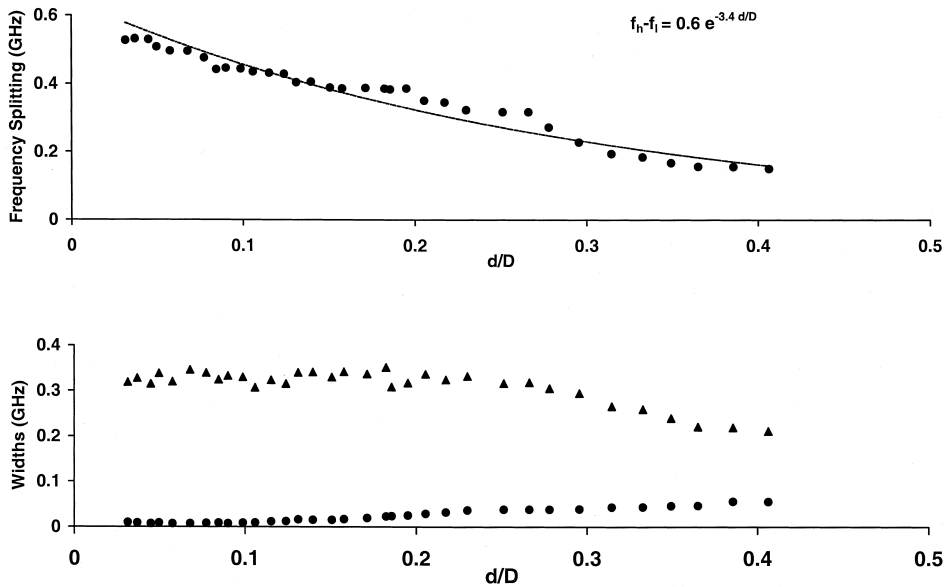


Fig. 3. (Top) Doublet frequency splitting Δf and (Bottom) Widths γ_l, γ_h versus distance d for the two-disc proximity resonance around 9.45 GHz. The result of an exponential fit in the region $\kappa_r d \geq 1$ is also shown.

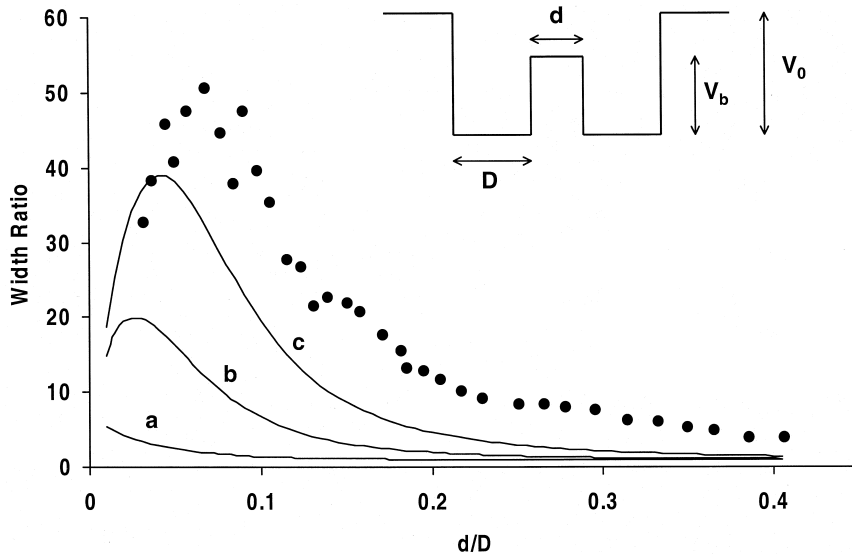


Fig. 4. Ratio between symmetric and antisymmetric width versus distance d/D for experimental data (dots) and various implementations of the double-well potential shown in the inset. Units where $\hbar = 2m = 1$ have been chosen. $V_0 = 900$ in units D^{-2} and $V_1/V_0 = -3 \times 10^{-2}$, $V_2/V_0 = -3 \times 10^{-6}$. The barrier height is $V_b/V_0 = 1$ (a), $1/4$ (b), $1/9$ (c).

coupling. The latter is known to take contributions only from the unperturbed evanescent fields $E_z(\mathbf{r}) = \cos(k_z z)E_z(x, y)$ of each resonator [3]. Experimentally, the antisymmetric mode is found to be able to store an extra amount of electromagnetic energy compared to the symmetric one [4]. This takes place through an amplification of field components (e.g., E_z, H_z), which do not contribute to power dissipation, whereby the higher observed Q .

The fact that the stabilisation of the antisymmetric mode only manifests at small separations suggests to picture the phenomenon in terms of a *collective* effect arising when two discrete states (the bound single-disk modes) are coupled to each other and, in addition, to a *common* environment (the metal) that renders them unstable. Similar effects are found in quantum physics, where they require an appropriate modification of the standard Weisskopf–Wigner decay theory [6]. The correspondence between electromagnetic (EM) and quantum mechanical (QM) systems is well established for stationary problems [7]. In particular, taking into account the boundary conditions at the dielectric surface, the component $E_z(x, y)$ in the waveguide plays the role of the wavefunction ψ in a 2D quantum mechanical system, the dielectric

medium corresponding to a square potential well at a *fixed* energy [12]. Accordingly, the two-disk system maps into a 2D tunneling problem. In the presence of losses, the damping of the EM field amplitude is usually accounted through a *complex* frequency $f - i\gamma/2$ whose imaginary part provides the time decay rate [3]. Despite the fact that, due to the different time-dependent equations of motion, the correct mapping to complex energies of quantum unstable states is a nonlinear relation of the form $(f - i\gamma/2)_{\text{EM}}^2 \Leftrightarrow (\varepsilon - i\gamma/2)_{\text{QM}}$, a QM configuration which is stabilized against decay will still be mapped into an EM non-decaying state.

A simple argument supporting the stability of the antisymmetric state goes as follows. Let $|L\rangle, |R\rangle$ denote degenerate ket states localized in the left, right well respectively. The two levels are coupled to each other by a tunneling perturbation of the form $H_T = -T[|L\rangle\langle R| + |R\rangle\langle L|]$, $T > 0$, and to a common continuum of states with a strength W_L, W_R . It is possible to show that the coupling to the environment mediates an extra interaction between the two discrete states, which strongly affects the decay properties of the combined system and thereby its spectral response [8]. If $W_L = W_R$, one predicts that

the symmetric combination $[|L\rangle + |R\rangle]$ corresponds to a Lorentzian resonance line at frequency f_S , whose width is twice larger than the width of each single level coupled to a continuum of the same strength, while the antisymmetric combination $[|L\rangle - |R\rangle]$, found at frequency f_A , is completely stabilized. This behavior can be regarded as the counterpart of Heller's predictions for the proximity effect based on a point scatterer model [11]. It is worth mentioning that the overall effect of the extra interaction indicated above is to dress the tunneling matrix element T with an additional *imaginary* part.

The actual situation, where the width of the antisymmetric mode is limited by the dielectric losses, would be more adequately modeled by invoking two distinct environments. To complement the previous analysis, we also explored a phenomenological description of dissipation in terms of an effective non-hermitian Hamiltonian [9]. By introducing a drastic approximation, we only consider an effective 1D potential projected along the horizontal symmetry axis¹. Within the theory of multidimensional tunneling, this is supported by the fact that the interaction is dominated by instanton orbits between the two centers [5]. Thus, we use a potential energy function of the form $V_{\text{pot}}(x) = V_0 + iV_1$ (for $|x| > d/2 + D$), iV_2 (for $d/2 < |x| < d/2 + D$), $V_b + iV_1$ (for $|x| < d/2$), all parameters being real numbers. The imaginary terms iV_1, iV_2 account for the losses outside and inside the double-well region respectively. We assume $|V_{1,2}|/V_{0,b} \ll 1$.

The real part of $V_{\text{pot}}(x)$ is depicted in Fig. 4 (inset). We allow for the possibility of a height barrier $V_b \neq V_0$ to effectively include corrections arising from the 2D nature of the problem. The presence of extra-contributions to the tunneling interaction, which are lost in the 1D model, is simulated by a more transparent barrier. The noninteracting limit corresponds to $d \rightarrow \infty$. For finite d , even and odd states are generated, with eigenenergies $E_P = \varepsilon_P - i\gamma_P/2$, $P = S, A$. If $\kappa = \kappa_r + i\kappa_i$ denotes the inter-well wave vector, each complex eigenvalue has a structure involving exponentials $e^{-\kappa_r d}$, convoluted with oscillating functions of $\kappa_i d$ whose details de-

pend on the state $|L\rangle, |R\rangle$ [4]. The signature of tunneling shows up through the exponential dependence of the energy splitting, $\Delta\varepsilon = \varepsilon_A - \varepsilon_S \approx e^{-\kappa_r d} F(\kappa_i d)$. The oscillatory terms in F are responsible for a ‘‘rippled’’ structure of the splitting decay, which is apparent in the data (Fig. 3(a)).

The transcendental equations determining E_A and E_S have been solved numerically for different sets of parameters with both $V_b = V_0$ and $V_b < V_0$ and the results compared with the experimental ones [4]. The leading exponential decay of the energy separation and the asymmetric small-distance splitting of the widths are correctly predicted. However, we find a major difference between the two models in their capability to reproduce the exceedingly large ratio between the widths. In the simulations with $V_b = V_0$, we were unable to reach ratios larger than 4, regardless of the values of the parameters V_1, V_2 , mainly affecting the absolute range of the widths. This order of magnitude is in agreement with independent results on symmetry splittings of resonances due to semiclassical creeping orbits [10]. In the reduced-height configuration, the ratio γ_s/γ_a can be controlled over a broad range (up to 50) by varying V_b/V_0 . Some representative behaviors are summarized in Fig. 4 for barrier opacity $V_b/V_0 = 1, 1/4, 1/9$. Maximum stability of the antisymmetric mode is reached at an intermediate distance d , corresponding to the γ_s/γ_a -peak value. The better qualitative agreement attainable with the reduced-height model suggests that *dimensionality* effects also play a key role in the experiment.

In order to confirm the conclusion that a judicious use of symmetry leads to a Q -sharpening, we carried out experiments on a 3-disk system, with the disks placed at the vertices of an equilateral triangle. Resonance triplets are observed. In analogy to the 2-disk system, we predict that modes transforming antisymmetrically with respect to reflections in each of the vertical mirror planes show enhanced stability against dissipation [4]. We focus on a single-resonator mode with $m = 3$ at 10.8 GHz, whose behavior is displayed in Fig. 5. A very sharp component at intermediate frequency is clearly seen. The Q factor is increased by roughly a factor 20 compared to the original one. The sharpening effect turns out to be very sensitive against symmetry-breaking effects. The influence of a geometric symmetry-breaking has been

¹ Such a procedure amounts to replacing the physical circular resonators with infinitely extended dielectric regions.

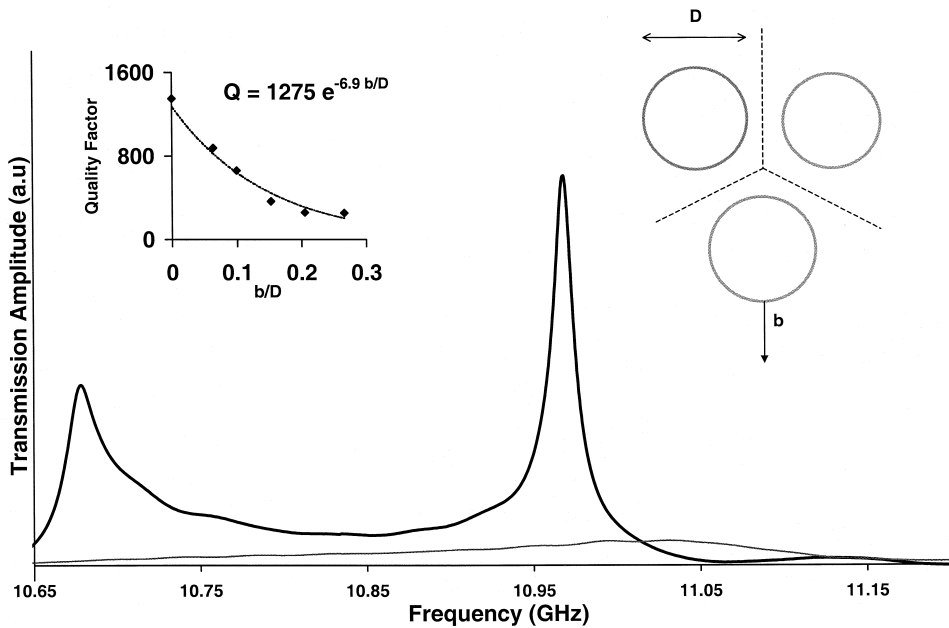


Fig. 5. Resonance triplet for three coupled resonators operating in the hybrid (3,1,1) mode at 10.8 GHz. (Inset) Quality factor of the sharp component as a function of the symmetry-breaking parameter b/D .

studied by shifting one of the disks by b along the bisectrix of the triangle. It is evident from Fig. 5 (inset) that the sharp resonance is dramatically affected, the Q factor being exponentially degraded. No sensible change is observed for any of the broad resonances.

In the spirit of the original definition by Heller [11], our observations indicate that interesting proximity phenomena arise in the spectral response of nearby systems. Proximity resonances have been recently detected in the scattering of a TEM electromagnetic mode in a parallel-plate waveguide [12]. Despite some superficial similarity with the present work, it is essential to realize that our experiments probed a completely different regime of the microwave field, where direct evanescent-wave coupling between *bound* modes rather than *scattering* resonances from two dielectrics illuminated by the same wave field were investigated. In particular, the power law behavior characterizing scattering states [11] should be contrasted with the exponential dependences that are intrinsically associated with tunneling. Thus, bound-state proximity resonances form a novel complementary manifestation of a similar

physical phenomenon, whose detailed understanding poses new challenges to both numerical simulations and semi-classical treatments.

Our results have a variety of implications. First, the tunneling interaction in a 2D integrable potential has been probed sensitively. There is no difficulty, in principle, to extend the experimental work to chaotic potentials where important results on complex periodic orbits theory have been obtained [5]. Second, the experiments demonstrate how symmetry properties can be usefully exploited to protect a system against the effects of its environment. This general mechanism provides a unifying explanation for proximity resonances, regardless of the unbound (scattering) or bound (confined) nature of the wave field. Third, our work can be related to recent observations of the symmetry splitting between optical modes in photonic molecules [13] where, however, the behavior of widths was not addressed. It is conceivable that some counterpart of proximity phenomena may be relevant on the mesoscopic scale as well. From the practical perspective, the possibility of symmetry-based Q -amplification in electromagnetic or photonic structures represents another exciting area of

applications. Finally, the electromagnetic phenomenon evidenced here displays intriguing similarities with concepts investigated in the context of quantum dissipative processes [6,14]. The possibility of establishing some mapping between the electromagnetic and quantum realm even in the presence of dissipative mechanisms would clearly open up a fruitful arena of interchange and deserves further investigation.

Acknowledgements

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